Sakharov`s Extra Timelike Dimensions and Hawking`s Chronology Protection Principle

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PREDICTIONS

- **Black hole production at CERN's Large Hadron Collider (LHC)**

- **Wormhole/time machine production at LHC**

Aref`eva, I.V.
Int.J.Geom.Meth.Mod.Phys.5(2008)641-651.
Mironov, Morozov, Tomaras.

Assumptions

- Classical geometric cross-section
- Extra dimensions at TeV
- "Exotics" : Dark energy, Casimir, non-minimal coupling, ghosts, extra timelike dimensions,…

Extra Timelike Dimensions

Sakharov hypothesis (1984):

the multiverse can contain spacetimes with different signatures of the metric including extra timelike dimensions.

Sakharov`s proposal:
summation over various signatures

$$\sum \int \exp\left\{ \int R\sqrt{g}\,dx \right\} Dg$$

Problems

In such spacetimes there are ghosts, tachyons and also there are closed timelike curves (CTC), i.e. time machines.

Too bad?

Solutions of D=11 SG

- **A class of vacuum solutions is obtained for D= 11 supergravity with a vanishing cosmological constant. In particular, there exists a solution**

  with an SO(4) X U(1) gauge group, and without ghosts and tachyons in the low-energy limit of the effective four-dimensional theory.

  Aref`eva, Dragovich, I.V. (1986)

Conditions for B

Massless ghosts are absent in the 4 dim theory if the following conditions are satisfied for internal manifold B:

B has no Killing vector fields.

If antisymmetric fields of the rank n are present then all odd Betti numbers

$$b_k(B) = 0, k \leq n.$$

Chronology Protection

Hawking conjectured that the laws of physics are such as to prevent time travel on all but sub-microscopic scales.

What could be a specific mechanics which would prevent closed timelike curves (CTC) from being formed?

Chronology Protection

Time machines violate the standard causality condition.

"It seems that there is a Chronology Protection Agency which prevents the appearance of CTC and so makes the universe safe for historians"

Hawking, Phys.Rev. (1992)

Outlook:

What is time?

- TIME MACHINES
- Extra Timelike Dimensions
- Nonglobally Hyperbolic Manifolds
- Chronology Protection
- QFT and CTC
- Cauchy Problem and CTC

What is Time?

- **Psychological time**
- **Biological time**
- **Physical time**
- **Mathematical time** (real numbers)
- **Different time scales. P-adic numbers.**

St. Augustine's Confessions:

"What then is time? If no one asks me, I know: if I wish to explain it to one that asketh, I know not."

Time

- whether time exists when nothing is changing;
- what kinds of time travel are possible;
- irreversibility problem;
- whether there was time before the Big Bang;
- whether tensed or tenseless concepts are semantically basic;
- what are the neural mechanisms that account for our experience of time.

…………………………………………………………..

Time Machine. Definition

- Spacetime: (M,g), M – manifold, g – metric.
- Einstein equations for g. (?)

- **Time machine is a region of space-time (M,g) that has a closed timelike curve (CTC).**

- CTC suggests the possibility of time travel with its well known paradoxes

- Example: time is circle.

CAUSALITY

Traversable wormholes/time machines contain small spacetime regions with closed timelike curves (CTC) which violate the standard causality condition.

Causality

- Cauchy problem. Global hyperbolic: $R \times \Sigma^3$
- Causality in QFT $$\frac{\delta}{\delta g(x)}\left(\frac{\delta S}{\delta g(y)} S^*\right) = 0, \quad x \le y$$

  Bogoliubov, Shirkov

- Local commutativity:

  LSZ, Whightman,… $$[\Phi(x), \Phi(y)] = 0, \quad (x-y)^2 > 0$$

  Bogoliubov, Tavkhelidze, Vladimirov,

- Locality in string theory: Gross, Veneziano, Susskind, ‘t Hooft, Giddings,…
- Nonlocality at the Planck scale: Bronstein, Wheeler, Blokhintzev, Markov, ‘t Hooft, p-adic space-time

Time Travel?

- **Grandfather Paradox**
- **Information Paradox**
- **Bio**
- K. Godel (1949)

General Relativity and Chronology

- **In GR one cannot simply *assert* that chronology is preserved, and causality respected, without doing considerable additional work.**
- **The essence of the problem lies in the fact that the Einstein eqs of GR are local equations, relating some aspects of the spacetime curvature at a point to the presence of stress-energy at that point.**
- **"In the small" GR respects all of the causality constraints of special relativity, but GR does *not* provide any natural way of imposing *global* constraints on the spacetime**
- **Without imposing additional principles along GR is completely infested with time machines**

Paradoxes generated by the possibility of time travel

There are two broad classes of paradox generated by the possibility of time travel

- **Grandfather paradoxes: Caused by attempts to "change the past",**
  and so modify the conditions that lead to the very existence of the
  entity that is trying to "modify the timestream".

- **Information paradoxes: bring information to past.**

Proposals

- **Make radical alterations to our worldview to incorporate at least some versions of chronology violation and "time travel".**

- **Permit constrained versions of closed timelike curves**

- **Incorporate quantum physics to intervene and provide a universal mechanism for preventing the occurrence of closed timelike curves.**

"Chronology Protection Conjecture"

Hawking

- There are long debate concerning such principles.
- Several people participated in these discussions.

Wheeler, Tipler, Thorne, Gott, Visser, …

Hawking, Deser, Jackiw, ‘t Hooft, …

“Chronology Protection Conjecture“

- It was suggested that large values of expectation value of the energy-momentum tensor occur when one has CTCs. If one fed this energy-momentum tensor into the Einstein eqs. it could prevent one from creating a TM.

- Or divergences in the energy-momentum tensor occur. These divergences may create space-time singularities which prevent one from traveling through to the region of CTC

Hawking's "chronology protection conjecture“

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G < T_{\mu\nu} >$$

$$G_R(x,y) = \sum_{\gamma \neq \gamma_0} \frac{\Delta_\gamma(x,y)^{1/2}}{4\pi^2} \{ \frac{1}{\sigma_\gamma(x,y)} +$$

$$\upsilon_\gamma(x,y) \ln|\sigma_\gamma(x,y)| + w_\gamma(x,y) \}$$

Hadamard form

$$< T_{\mu\nu} >_R = \lim_{x \to y} D_{\mu\nu}(x,y,\gamma_0) G(x,y)$$

Hawking's "chronology protection conjecture“

$$< T_{\mu\nu} >_R = \sum_{\gamma \neq \gamma_0} \frac{\Delta_\gamma (x,x)^{1/2}}{\sigma_\gamma (x,x)^2} t_{\mu\nu}(x) + \ldots$$

Theorem (Kay,Radzikowski,Wald). There are points on the chronology horizon where the two-point functions is not of Hadamard form

Black Holes in Collisions

- A possibility of production in ultra-relativistic particle collisions of some objects related to a non-trivial space-time structure is one of long-standing theoretical questions
- Gravitational radiation in collision of two classical ultra relativistic particles was considered by D'Eath and Payne (1978,1992) and
- the mass of the assumed final BH also has been estimated

- In 1987 Amati, Ciafaloni, Veneziano and 't Hooft conjectured that in string theory and in QG at energies much higher than the Planck mass BH emerges.

- Aichelburg-Sexl shock waves to describe particles,

  Shock Waves ------ > BH
- Colliding plane gravitation waves to describe particles

  Plane Gr Waves ----- > BH I.Arefe'eva, Viswanathan, I.V., 1995

Quantum Gravity

Transition amplitude: **Two particles** ➡ **black hole / wormhole**

$$< h'', \phi'', \Sigma'' | h', \phi', \Sigma' >= \int_{\text{Sum over topologies}} \exp\{\frac{i}{\hbar} S[g,\Phi]\}\, dg\, d\Phi,$$

$$\Sigma'': h_{ij}'', \phi'' \quad ; \quad \Sigma': h_{ij}', \phi',$$

Wheeler- de Witt formalism

$$g|_{\Sigma''} = h'', \Phi|_{\Sigma''} = \phi''; \;\; g|_{\Sigma'} = h', \Phi|_{\Sigma'} = \phi'$$

AVV
NP, B452,1995

No explicit time. **Summation over topologies**

Wave functions: $\Psi_{\Sigma'}[h', \phi']$ *two particles*

$\Psi_{\Sigma''}[h'', \phi'']$ *black hole, whormhole,...*

To speak about the production of black holes in quantum theory one should have a notion of a quantum BH as a state (pure or mixed) in some Hilbert space.

Summation over topologies

$$< h'', \Sigma'' | h', \Sigma' > = \int\limits_{\text{sum over topologies}} \exp\{\frac{i}{\hbar} S[g, \Phi]\} \, dg \, ,$$

$$g|_{\Sigma''} = h''; \quad g|_{\Sigma'} = h'$$

No coupling constant to suppress-out channels with nontrivial topology

Summation over topologies

$$<h'',\Sigma''|h',\Sigma'>= \int_{\text{sum over topologies}} \exp\{\frac{i}{\hbar}S[g,\Phi]\}\, dg\ ,$$

$$g|_{\Sigma''}=h'';\ \ g|_{\Sigma'}=h'$$

Theorem: Geroch, Tipler
Topology-changing spacetimes must have CTC

Gammon and Perelman theorem (Poincare conjecture):
If asymptotically flat spacetime has a Cauchy surface with a nontrivial topology, then the **spacetime is geodesically incomplete**

BLACK HOLE PRODUCTION

- Collision of two fast point particles of energy *E*.
- BH forms if the impact parameter *b* is comparable to the Schwarzschild radius r_s of a BH of mass *E*.
- The Thorne's hoop conjecture gives a rough estimate for classical geometrical cross-section

$$\sigma(1+1 \rightarrow \mathrm{BH}) \sim \pi \, r_S^2$$

BLACK HOLE PRODUCTION

Arkani-Hamed,
Dimopoulos, Dvali,
Antoniadis, 1998
Hierarchy problem

However if the fundamental Planck scale of QG is of the order of few TeVs then there is an exciting possibility of production of BHs, branes, K-K modes in proton-proton collisions at the LHC.

$$M_{Pl}^{2} = M_{D}^{2}\left(\frac{M_{D}}{M_{c}}\right)^{D-4}$$

Giudice, Rattazzi, Wells;
Banks, Fischler; I.Aref'eva, Ringwald,Tu;
Giddings,Thomas; Dimopoulos,
Landsberg; Kaloper; Cavaglia,Cardaso,…
Gingrich;Yoshino, Rychkov, Volobuev,…
I.Aref'eva, I.V.; Mironov, Morozov,
Tomaras….

Pros and cons of signatures of BH production

Mende, Randall,…

Catalyze of BH production due to an anisotropy

Dvali, Sibiryakov

Modification of the Newton law

$$F = \frac{G_{Newton}}{r^2} m_1 m_2 \quad \Rightarrow \quad F = \frac{G_{Newton}}{r^2} m_1 m_2 \qquad for\ r \geq L_c$$

$$F = \frac{V_n}{r^n} \frac{G_{Newton}}{r^2} m_1 m_2 \qquad for\ r \leq L_c$$

Wormholes

- Lorentzian Wormhole is a region in spacetime in which 3-dim space-like sections have non-trivial topology.

- By non-trivial topology we mean that these sections are not simply connected

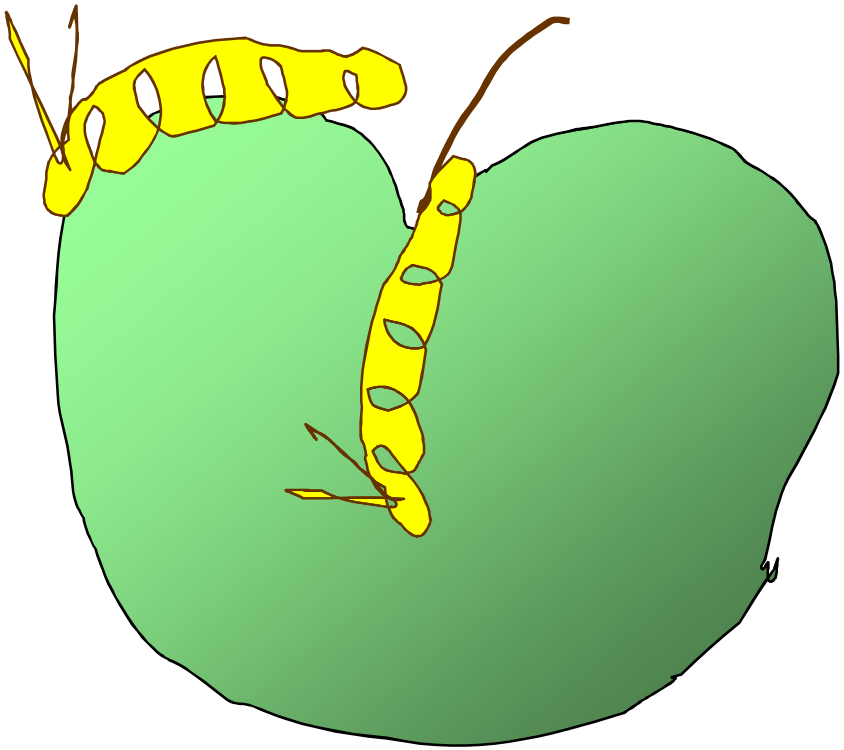
- In the simplest case a WH has two mouths which join different regions of the space-time.

- We can also imagine that there is a thin handle, or a throat connected these mouths.
- Sometimes people refer to this topology as a 'shortcut' through out spacetime

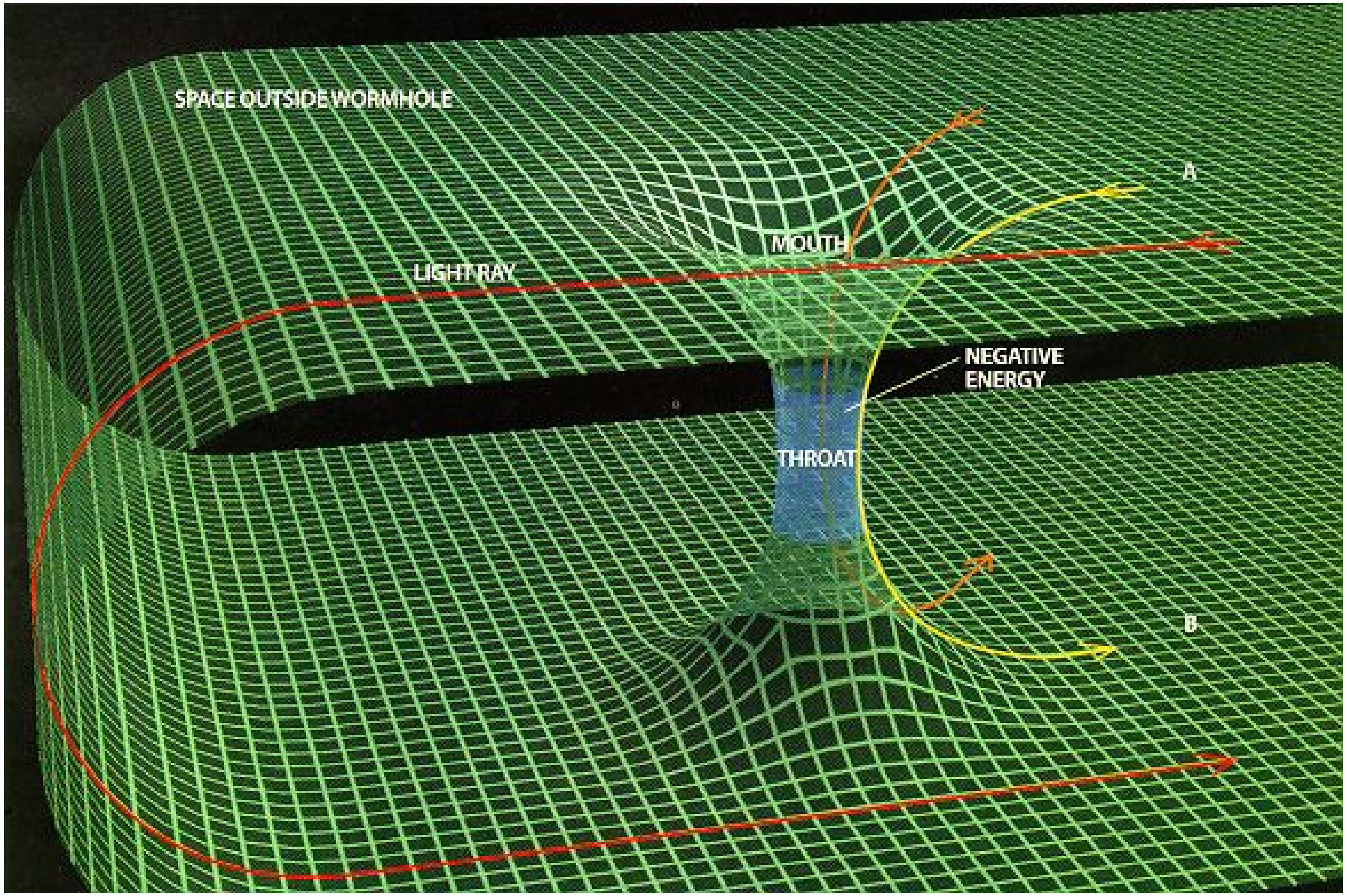
WHs in astrophysics, Kardashov, Novikov,…

Wormholes

- The term WH was introduced by J. Wheeler in 1957
- Already in 1921 by H. Weyl (mass in terms of EM)
- The name WH comes from the following obvious picture.



The worm could take a shortcut to the opposite side of the apple's skin by burrowing through its center, instead of traveling the entire distance around.



The traveler just as a worm could take a shortcut to the opposite side of the universe through a topologically nontrivial tunnel.

Wormholes

- H.Weyl. Solution was found by Einstein and Rosen in 1935 (E-R bridge)
- There are many wormhole solutions in GR.
- A great variety of them! With static throat, dynamic throat, spinning, not spinning, etc
- Schwarzschild WHs (E-R bridges)
- The Morris-Thorne WH
- The Visser WH
- Higher-dimensional WH
- Brane WH

Traversable Wormholes

Morris, Thorne, Yurtsever, Visser,..

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r^2}} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

Traversable Lorentzian wormholes (WH)

Ellis's WH

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{r_0^2}{r^2}} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

$$T_{tt} = \rho(r)$$
$$T_{rr} = p_r(r)$$
$$T_{\vartheta\vartheta} = T_{\varphi\varphi} = p(r)$$

$$\rho(r) = -p_r(r) = - p(r) = -\frac{r_0^2}{8\pi\, r^4}$$

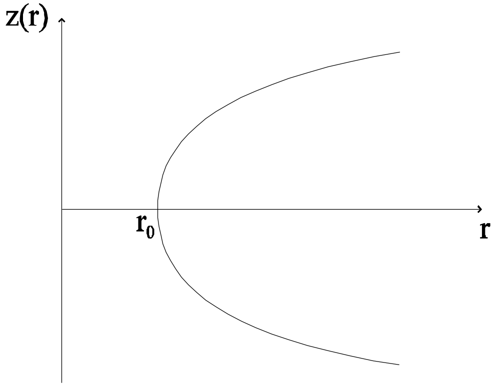
$$w = -1$$

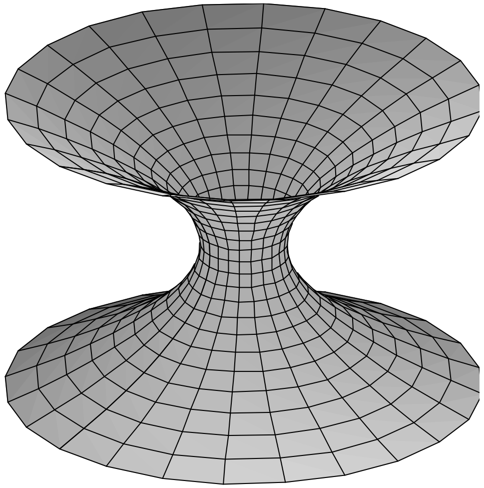
Geometry. $t = const, \quad \varphi = const$

$$ds^2 = +\frac{dr^2}{1 - \frac{r_0^2}{r^2}} + r^2 d\vartheta^2 = dx^2 + dy^2 + dz^2 \,|_{2-\mathrm{dim}}$$

$$z = z_\pm(r), \quad r > r_0 > 0,$$

$$\frac{dz_\pm(r)}{dr} = \pm\frac{r_0}{\sqrt{r^2 - r_0^2}}$$





Bronnikov,..

Traversable Wormholes

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

$$r_0 \le r < R$$

For asymptotically flat WH

WH throat $b(r_0) = r_0$

$$R = \infty$$

Absence of the event horizon $b'r - b < 0$

$$\rho + p_r = \frac{1}{M_{Pl}}\left(\frac{b'r - b}{r^3} + 2(1-\frac{b}{r})\frac{\Phi'}{r}\right)$$

The embedding condition together with the requirement of finiteness of the redshift function lead to the NEC violation on the WH throat

Higher Dimensional WHs

In the brane world scenario, where the Universe is considered as a 3-brane embedded in a D-dimensional bulk, the 4-dim Einstein equations contain the effective stress energy tensor

$$G_{\mu\nu} = \frac{1}{M_{Pl}^{2}} T_{\mu\nu}^{eff}$$

F.Lobo

This effective 4-dim stress energy tensor is a sum of the stress energy tensor of a matter confined on the brane and correction terms.

The correction terms arise from a projection of the D-dim. Einstein equations to the 4-dim space-time.

D=5 example

Shiromizu, Maeda, Sasaki

A relaxed condition appears due to corrections from the Weyl tensor in the bulk.
4-dim effective stress energy tensor violates the NEC, meanwhile the total 5-dim stress energy tensor does respect the NEC

D-dimensional WH Solution

$$r_0 = \gamma_{WH}(D)\frac{1}{M_D}\left(\frac{M_{WH}}{M_D}\right)^{\alpha_{WH}}$$

r_o *is the* radius of the throat

$$r_0 > r_S$$

Time Machine. Definition

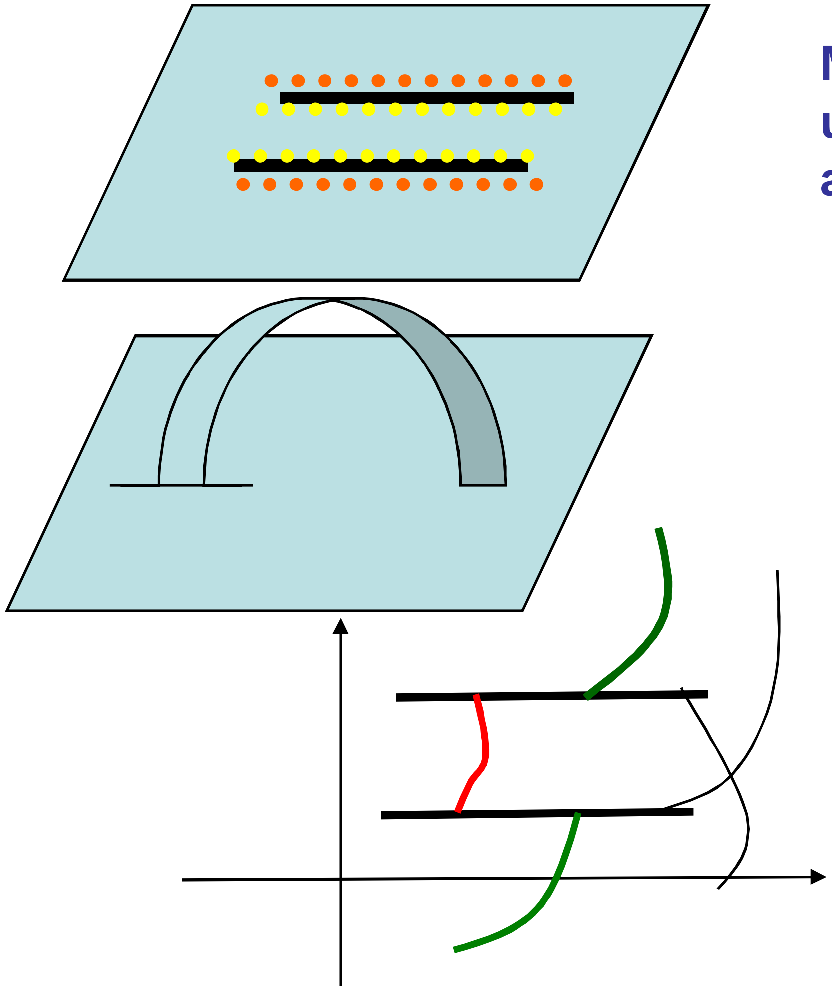
- **Time machine is a region of space-time (M,g) that has a closed timelike curve (CTC).**
- **CTC suggests the possibility of time-travel with its well known paradoxes**

CTC ⟷ Time Machine

Time Machine

Surgery in the Minkowski spacetime

Deutsch, 1991



This space contains timelike loops

Make two cuts and glue the upper edge of upper cut to the lower edge of the lower cut and vice verse,

So we get the plane with a handle.

It is convenient to draw the resulting spacetime still as M, and just to keep in mind the identification rules.

Q.: what could force the space-time to evolve into this construction instead of just remaining the Minkowski spacetime

Solutions of Einstein eqs. with Closed Timelike Curves (CTC) / Time Machine.

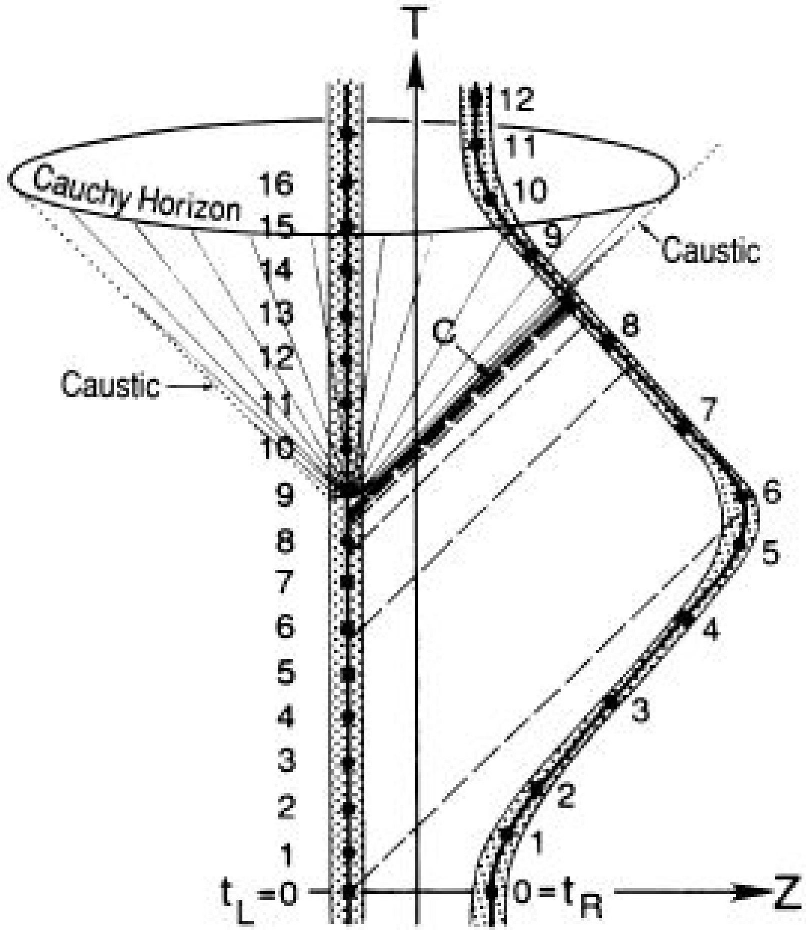
- **Godel's solution [1949]**
- **van Stockum-Tipler cylinder [1937, 1974];**
- Kerr solutions; **2 axially symmetric, stationary Kerrs**
- **Gott's time machine;**
- **Wheeler wormholes;**
- **Morris-Thorne-Yurtsever's TM**
- Ori's dust asymptotically-flat space-time
- Frolov, I.Novikov, Mensky,…

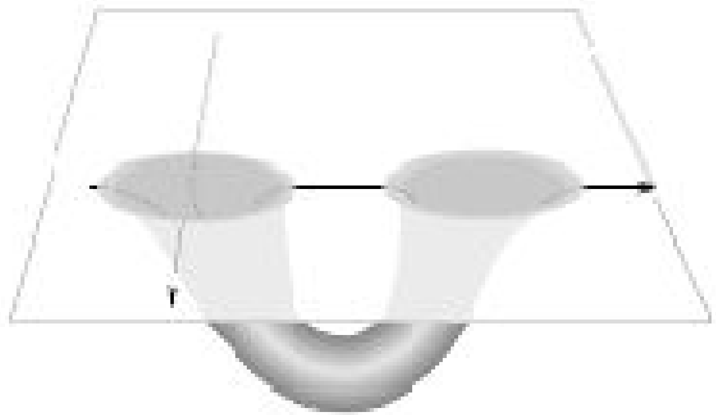
Mathematical solution of Grandfather paradox

Overcoming of the grandfather paradox:

There are spacetimes having CTC for which smooth, unique solutions to the scalar wave equation exist for constrained data on the Cauchy surface.

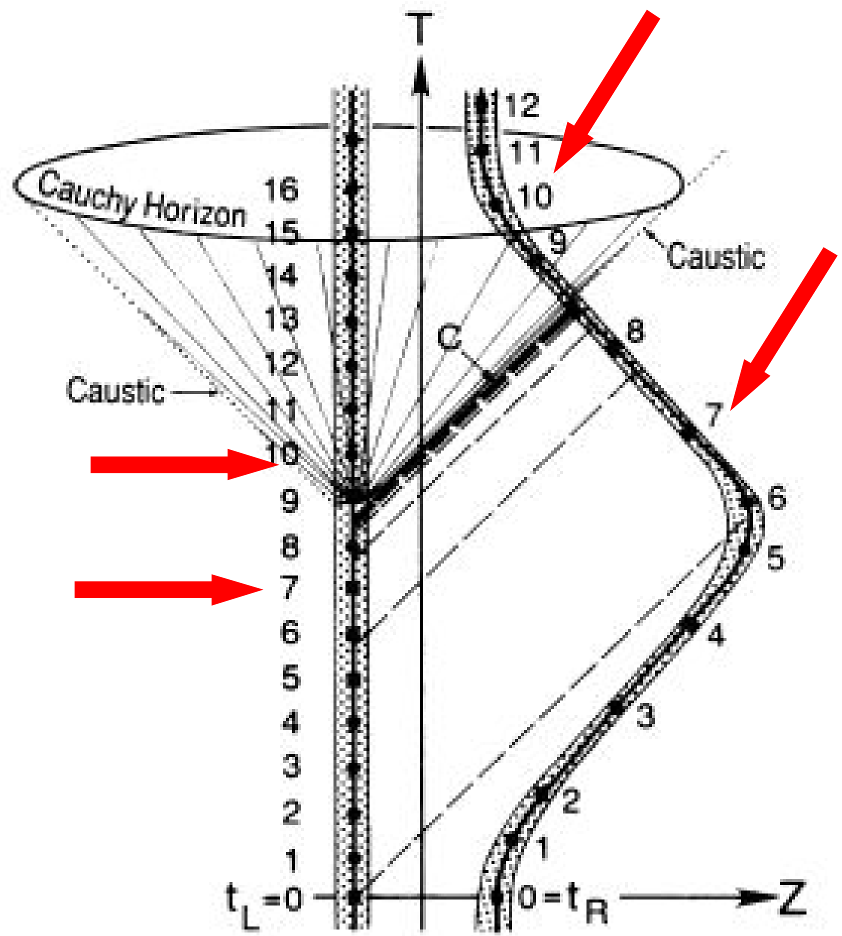
Morris, Thorne, Yurtsever Time Machine





Spacetime diagram for the conversion of a spherical, traversible wormhole into a TM.

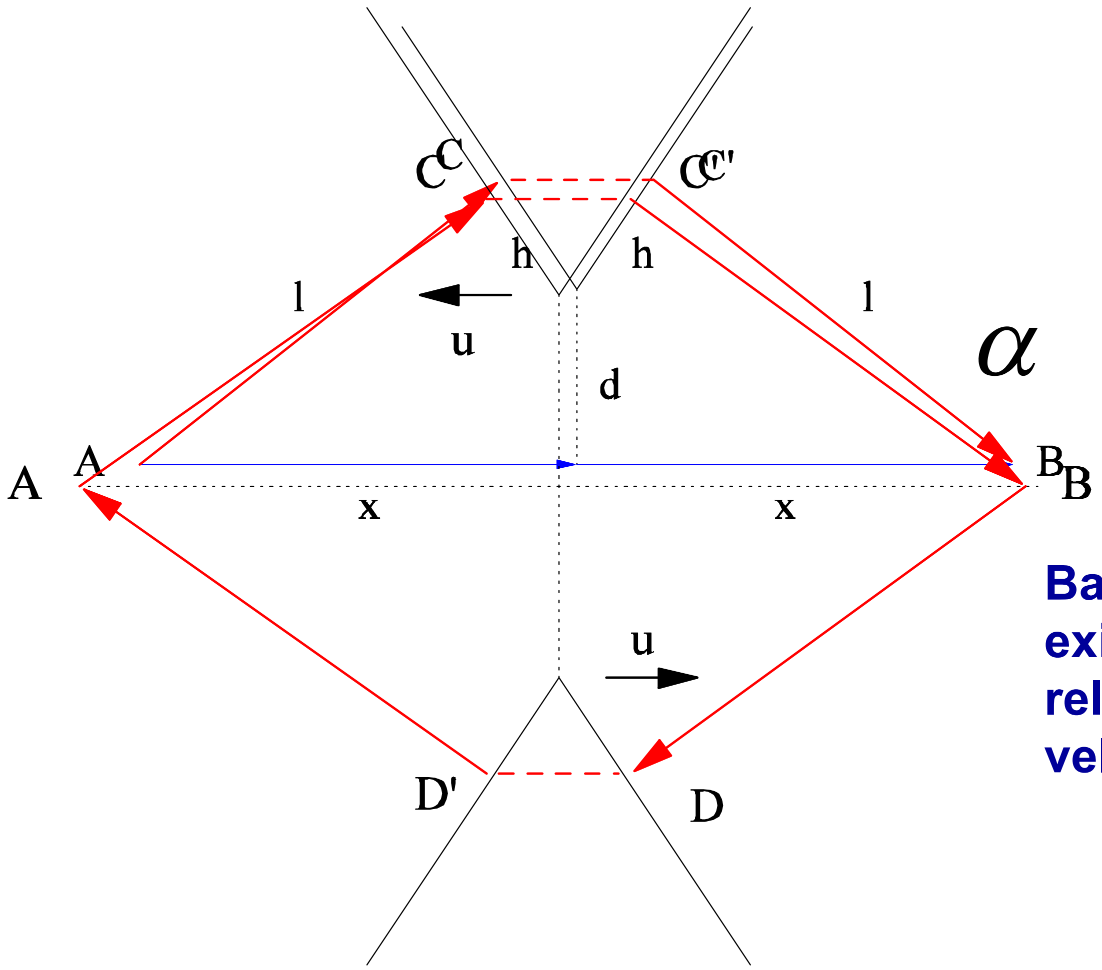
Spacetime diagram for the conversion of MT WH into a TM.



- At T=0 the WH's mouths are at rest near each other
- Later, the left mouth remains at rest while the right mouth accelerates to near-light speed, then reverses its motion and returns to its original location.
- This motion causes the right mouth to "age“ less than the left. Some of the identified points are causally related in the initial Minkowski stacetime.
- Two points marked as "7" at the right and left mouths are not causally related,

  but points "10" are causally related.

Gott's TM



Let compare two trajectories:
a direct path AB;
ACC'B path.

"wedge" signal arrives before a "direct" signal if l < x

Back-in-time paths across the wedge exist in a frame moving relative to the cosmic string with velocity

$$u > \cos\alpha$$

CTCs looping around the two string

Critics: Deser, R. Jackiw, G. 't Hooft

Quantum mechanics with CTC

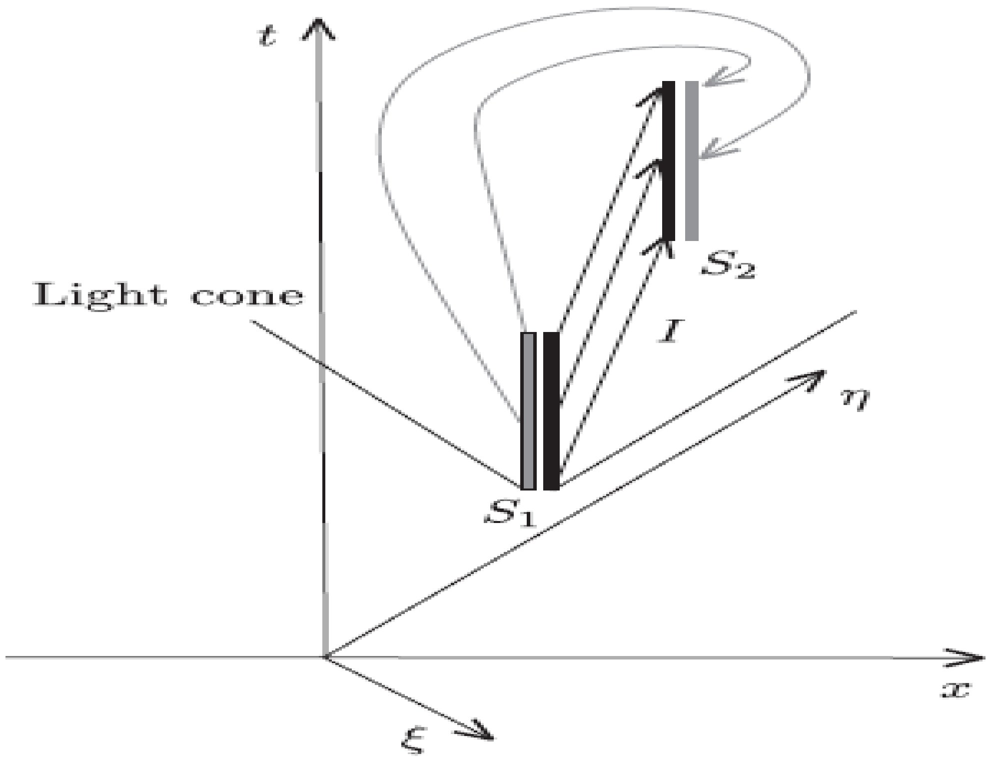
- Deutch, Politzer,…
- Unitary quantum evolution
- Klein-Gordon equation on (M,g) with CTC
- Hawking`s chronology protection

Solutions of the Wave Equation on the Nonglobally Hyperbolic Manifold

- Cauchy problem for the wave equation on the non-globally hyperbolic manifold (Minkowski plane with handle) containing CTC (time machine).
- Classical solution exists if and only if the initial date satisfy a set of constraints.
- Groshev, Gusev, Kurianovich, I.V.

  arXiv:0903.0741

Minkowski Plane with Hole



Wave Equation

$$u_{tt} - u_{xx} = 0, \;\; (x,t) \in \Omega$$

with initial conditions

$$\begin{aligned} u(x,0) &= \varphi(x)\,, \\ u_t(x,0) &= \psi(x)\,, \end{aligned}$$

$$\gamma_1 = \left\{(x,t) \in \mathbb{R}^2_+ | x = a_1, \; b_1 < t < b_1 + \ell\right\},$$

$$\gamma_2 = \left\{(x,t) \in \mathbb{R}^2_+ | x = a_2, \; b_2 < t < b_2 + \ell\right\}$$

Boundary Conditions

$$
\begin{aligned}
u(a_1 - 0, t) &= u(a_2 + 0, t + b_2 - b_1)\,, \\
u(a_1 + 0, t) &= u(a_2 - 0, t + b_2 - b_1)\,, \\
u_x(a_1 - 0, t) &= u_x(a_2 + 0, t + b_2 - b_1)\,, \\
u_x(a_1 + 0, t) &= u_x(a_2 - 0, t + b_2 - b_1),
\end{aligned}
$$

∎

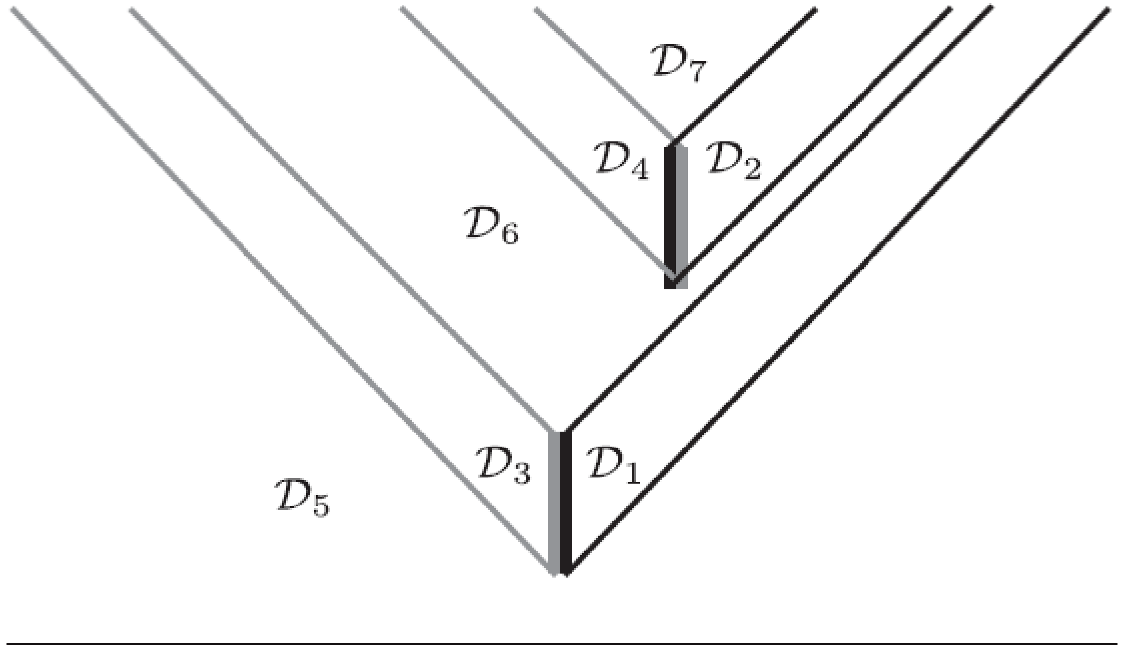


Figure 2: Domains $D_1,...,D_7$

$$u(x,t) = f(x - t + A_i) + g(x + t + B_i) + C_i$$

10 constraints

$$a = a_2 - a_1, \quad b = b_2 - b_1,$$

$$c_i = a_i - b_i, \quad d_i = a_i + b_i,$$

$$\varphi(c_2 - \ell) - \varphi(c_1 - \ell) + \int_{c_2-\ell}^{c_1-\ell} \psi(s)\, ds = \varphi(d_1) - \varphi(d_2) - \int_{d_1}^{d_2} \psi(s)\, ds,$$

$$\varphi(d_2 + \ell) - \varphi(d_1 + \ell) + \int_{d_1+\ell}^{d_2+\ell} \psi(s)\, ds = \varphi(c_1) - \varphi(c_2) - \int_{c_2}^{c_1} \psi(s)\, ds.$$

$$
\begin{aligned}
\varphi^{(i)}(c_1) - \psi^{(i-1)}(c_1) &= \varphi^{(i)}(c_2) - \psi^{(i-1)}(c_2) \\
\varphi^{(i)}(d_1) + \psi^{(i-1)}(d_1) &= \varphi^{(i)}(d_2) + \psi^{(i-1)}(d_2) \\
\varphi^{(i)}(c_1 - \ell) - \psi^{(i-1)}(c_1 - \ell) &= \varphi^{(i)}(c_2 - \ell) - \psi^{(i-1)}(c_2 - \ell) \\
\varphi^{(i)}(d_1 + \ell) + \psi^{(i-1)}(d_1 + \ell) &= \varphi^{(i)}(d_2 + \ell) + \psi^{(i-1)}(d_2 + \ell)
\end{aligned}
$$

Theorem: Solution to the wave equation

$$u(x,t) = u_i(x,t) \quad if \quad (x,t) \in D_i,\ i = 1, \ldots, 7,$$

where

$$\begin{aligned}
u_1(x,t) &= f(\eta + c) + g(\xi) + f(c_1) - f(c_2) \\
u_2(x,t) &= f(\eta - c) + g(\xi) + g(d_1) - g(d_2) \\
u_3(x,t) &= f(\eta) + g(\xi + d) + g(d_1) - g(d_2) \\
u_4(x,t) &= f(\eta) + g(\xi - d) + f(c_1) - f(c_2) \\
u_5(x,t) &= f(\eta) + g(\xi) \\
u_6(x,t) &= f(\eta) + g(\xi) + g(d_1) - g(d_2) + f(c_1) - f(c_2) \\
u_7(x,t) &= f(\eta) + g(\xi);
\end{aligned}$$

..

$here\ \xi = x + t,\ \eta = x - t,$

$$f(x) = \frac{1}{2}\left[\varphi(x) - \int_{x_0}^{x} \psi(s)\,ds\right]$$

$$g(x) = \frac{1}{2}\left[\varphi(x) + \int_{x_0}^{x} \psi(s)\,ds\right].$$

Another form of the solution

$$u(x,t) = u^D(x,t) + U(x-a_1, t-b_1) - U(x-a_2, t-b_2),$$

where

$$u^D(x,t) = \frac{1}{2}[\varphi(x+t) + \varphi(x-t)] + \frac{1}{2}\int_{x-t}^{x+t} \psi(s)ds,$$

$$U(x,t) = \frac{1}{2}\theta(t-|x|)\int_0^{t-|x|} \omega(\tau)\, d\tau - \theta(t-|x|)\frac{\operatorname{sign} x}{2}\nu\left(t-|x|\right),$$

$$\omega(t) = \theta(t)\theta(\ell - t) \cdot \left(u_t^D(a_2, b_2+t) - u_t^D(a_1, b_1+t)\right),$$

$$\nu(t) = \theta(t)\theta(\ell - t) \cdot \int_0^t \left(u_x^D(a_2, b_2+\tau) - u_x^D(a_1, b_1+\tau)\right) d\tau.$$

Conclusions

- **TeV Gravity opens new channels – BHs, WHs**
- **Mini time machines (traversable wormholes) could be produced at LHC**
- **Important question on possible experimental signatures of mini time machines at LHC requires further explorations**
- **Hope to understand better: What is time?**