Renormalization Group Relations and Model-Independent Searches for Z' within the Data of LEP Experiments

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Main goals:

1) To consider Z' boson independently of the specific features and details of a theory beyond the SM.

What can we generally expect for its mass and couplings from the theoretical point of view?

Is it possible to derive some model-independent constraints on Z' couplings, following from some general necessary theoretical principles?

2) To show the crucial role of the theoretical knowledge in order to extract maximal information about Z' from the experimental data (the final LEP2 data).

Waiting for Z' effects in some arbitrary (traditionally) chosen observable is just a hope for a lucky accident (squandering of limited experimental statistics). As a result, the only information on Z' from LEP collaborations is $m_{Z'} > 400 - 800$ GeV for a limited set of popular models beyond the SM.

Instead, we have found observables which are most sensitive to Z' As a result, 1-2σ hints of the particle are found.

Probably, we will not be completely blind in searching for Z' at the LHC.

3) To summarize the present experimental constraints on Z' parameters from the LEP data.

This summary is a basic guide for future searching for Z' at the LHC.

Outline

- General requirements to a model beyond the SM containing Z' (renormalizability, Z' decoupling, the SM is a subgroup of an unknown gauge group at high energies).
- Relations between Z' couplings (RG relations).
- Annihilation processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$
- Bhabha scattering process $e^+e^- \to e^+e^-$
- Many parameter fit of the LEP2 data on the leptonic processes
- Conclusion. Have we found traces of Z'?

General requirements to a model beyond the SM containing Z'

Renormalizability

- UV divergencies in radiative corrections reproduce the tree level structure. Dominating (tree-level) Z' interactions are of renormalizable type (potential$\times$current). Non-renormalizable type interactions are generated by loops and suppressed.
- Any scattering amplitude is invariant under the change of normalization point (satisfies the RG equation).

Decoupling

Asymptotics of Passarino-Veltman loop integrals: large logarithms of heavy mass reproduce the UV divergencies and washed out by the renormalization. At low energies the running of charges, masses and wave functions are governed by light particles inside loops. All the effects of heavy particle are suppressed by powers of heavy mass (heavy particle decoupling) [Appelquist-Carazzone theorem].

The SM is a subgroup of an unknown gauge group at high energies

Z' interactions to the SM gauge bosons at the tree level are due to the mixing, only.

1 Parametrization of the Z' couplings

Let us parametrize the fermion-vector interactions introducing the effective low-energy Lagrangian:
[Cvetic, Lynn (1987), Degrassi, Sirlin (1989); reviews, Leike (1999), Langacker (2008)]

$$\begin{aligned} L_f &= i \sum_{f_L} \bar{f}_L \gamma^\mu \left(\partial_\mu - \frac{ig}{2} \sigma_a W_\mu^a - \frac{ig'}{2} B_\mu Y_{f_L} - \frac{i\tilde{g}}{2} \tilde{B}_\mu \tilde{Y}_{f_L} \right) f_L \\ &+ i \sum_{f_R} \bar{f}_R \gamma^\mu \left(\partial_\mu - ig' B_\mu Q_f - \frac{i\tilde{g}}{2} \tilde{B}_\mu \tilde{Y}_{f_R} \right) f_R, \end{aligned} \quad (1)$$

where summation over all SM left-handed fermion doublets, leptons and quarks, $f_L = (f_u)_L, (f_d)_L$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood. Q_f denotes the charge of f in positron charge units, $\tilde{Y}_{f_L} = \mathrm{diag}(\tilde{Y}_{f_u}, \tilde{Y}_{f_d})$, and $Y_{f_L} = -1$ for leptons and $1/3$ for quarks.

Z' interactions with the scalar doublets can be parametrized in a model-independent way as follows,

$$L_{\phi} = \sum_{i=1}^{2} \left| \left(\partial_{\mu} - \frac{ig}{2}\sigma_a W_{\mu}^{a} - \frac{ig'}{2} B_{\mu} Y_{f_L} - \frac{i\tilde{g}}{2} \tilde{B}_{\mu} \tilde{Y}_{\phi_i} \right) \phi_i \right|^2 . \qquad (2)$$

In these formulas, $g, g', \bar{g}$ are the charges associated with the $SU(2)_L, U(1)_Y$, and the Z' gauge groups, respectively, σ_a are the Pauli matrices, $\tilde{Y}_{\phi_i} = \mathrm{diag}(\tilde{Y}_{\phi_{i,1}}, \tilde{Y}_{\phi_{i,2}})$ is the generator corresponding to the gauge group of the Z' boson, and Y_{ϕ_i} is the $U(1)_Y$ hypercharge.

The Yukawa Lagrangian can be written in the form

$$\begin{aligned} L_{Yuk.} &= -\sqrt{2} \sum_{f_L} \sum_{i=1}^{2} \Big(G_{f_{d,i}} [\bar{f}_L \phi_i (f_d)_R + (\bar{f}_d)_R \phi_i^{+} f_L] \\ &+ G_{f_{u,i}} [\bar{f}_L \phi_i^{c} (f_u)_R + (\bar{f}_u)_R \phi_i^{c+} f_L] \Big), \end{aligned} \qquad (3)$$

where $\phi_i^c = i\sigma_2 \phi_i^*$ is the charge conjugated scalar doublet.

Low energy parameters $\tilde{Y}_{\phi_{i,1}}$, $\tilde{Y}_{\phi_{i,2}}, \tilde{Y}_{L,f}, \tilde{Y}_{R,f}$ must be fitted in experiments. In most investigations they were considered as independent ones.

Are they independent under our assumptions about a theory beyond the SM?

All of them correspond to renormalizable type interactions. But how about the RG equation?

2 Renormalization group relations

What is RG relation?

Generally speaking, this is a correlation between low energy parameters of interactions of a heavy new particle with known light particles of the SM following from the requirement that full unknown yet theory extending SM is to be renormalizable.

Strictly speaking, RG relations are the consequence of two constituencies:

1) RG equation for a scattering amplitude;
A scattering amplitude f is independent of a choice of the normalization point (the RG equation):

$$Df = \left(\frac{\partial}{\partial \log \mu} + \sum_a \beta_a \frac{\partial}{\partial \hat{\lambda}_a} - \sum_{\hat{X}} \gamma_X \frac{\partial}{\partial \log \hat{X}} \right) f = 0, \tag{4}$$

where f accounts for as intermediate states either the light or heavy virtual particles of the full theory.

2) Decoupling theorem.

The decoupling theorem describes the rearrangement of series of perturbation theory including the modification of both the RG operator

$$D = \frac{d}{d\log\mu} = \frac{\partial}{\partial\log\mu} + \sum_a \beta_a \frac{\partial}{\partial\hat{\lambda}_a} - \sum_{\hat{X}} \gamma_X \frac{\partial}{\partial\log\hat{X}} \tag{5}$$

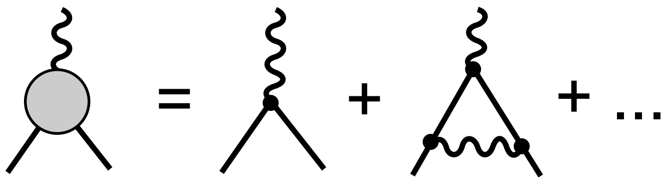
and an amplitude at the energy threshold Λ of new physics. Here, β_a- and γ_X-functions correspond to all the charges $\hat{\lambda}_a$ and fields and masses $\hat{X}$ of the underlying theory. As a result, no logarithms of heavy mass appear in observable quantities.

The standard usage of the RG equation is to improve the amplitude by solving this equation for the operator D calculated in a given order of perturbation theory.

However, to search for heavy virtual particles,
we will use Eq. (4) in another way.

First note that for any renormalizable theory, the RG equation is just identity, if f and D are calculated in a given order of loop expansion. In this case Eq.(4) expresses the well known fact that the structure of the divergent term coincides with the structure of the corresponding term in a tree-level Lagrangian.

For example, in massless QED, the tree-level plus one-loop one-particle-irreducible vertex function describing scattering of electron in an external electromagnetic field $\bar{A}$, $\Gamma = \Gamma^{(0)} + \Gamma^{(1)}$, is



If we calculate the RG operator in one-loop order

$$D = \frac{\partial}{\partial \log \mu} + \beta_e^{(1)} \frac{\partial}{\partial e} - 2\gamma_\psi^{(1)} - \gamma_A^{(1)}, \tag{6}$$

where $\beta_e^{(1)}$**,** $\gamma_A^{(1)}, \gamma_\psi^{(1)}$ **are the beta-function and the anomalous dimensions of electromagnetic and electron fields, respectively, and apply it to** Γ**, we obtain**

$$-\frac{\partial}{\partial \log \mu}\Gamma^{(1)} = \left(\beta_e^{(1)} \frac{\partial}{\partial e} - 2\gamma_\psi^{(1)} - \gamma_A^{(1)}\right)\ \Gamma^{(0)} + O(e^5). \tag{7}$$

Then, accounting for the values of

$$\beta_e^{(1)} = \frac{e^3}{12\pi^2}, \quad \gamma_A^{(1)} = \frac{e^2}{12\pi^2}, \quad \gamma_\psi^{(1)} = \frac{e^2}{16\pi^2} \tag{8}$$

and the factor e **in** $\Gamma^{(0)}$**, we observe that the first and the last terms in the r.h.s. cancel. Since** μ**-dependent term in** $\Gamma^{(1)}$ **is** $\Gamma_\mu^{(1)} = \frac{e^3}{16\pi^2} \log \mu^2$**, we see that Eq.(7) is identity in the order** $O(e^3)$**.**

Next important point is that in a theory with different mass scales the decoupling of heavy-loop contributions at the threshold of heavy masses, Λ, results in the following property:

the running of all functions is regulated by the loops of light particles.

Therefore, the β and γ functions at low energies are determined by the SM particles, only. This fact is the consequence of the decoupling theorem [Appelquist, Carrazone (1975); Collins, Wilczek, Zee (1978)].

The decoupling results in the redefinition of parameters at the scale Λ and removing heavy-particle loop contributions from RG equation [Bando, et al. (1993), Gulov, Skalozub (2000)]:

$$\begin{aligned} \lambda_a &= \hat{\lambda}_a + a_{\hat{\lambda}_a} \log \frac{\Lambda^2}{\mu^2} + b_{\hat{\lambda}_a} \log^2 \frac{\Lambda^2}{\mu^2} + \cdots, \\ X &= \hat{X} \left(1 + a_{\hat{\lambda}_a} \log \frac{\Lambda^2}{\mu^2} + b_{\hat{\lambda}_a} \log^2 \frac{\Lambda^2}{\mu^2} + \cdots \right), \end{aligned} \qquad (9)$$

where λ_a and X denote the parameters of the SM. They are calculated assuming that no heavy particles are excited inside loops.

The matching between both sets of parameters λ_a, X and $\hat{\lambda}_a$, $\hat{X}$ is chosen at the normalization point $\mu \sim \Lambda$,

$$\lambda_a|_{\mu=\Lambda} = \hat{\lambda}_a|_{\mu=\Lambda}, \qquad X_a|_{\mu=\Lambda} = \hat{X}_a|_{\mu=\Lambda}. \qquad (10)$$

The differential operator D in the RG equation is in fact unique; the apparently different D in both theories are the same!

Note that if a theory with different mass scales is specified one can freely replace the parameters λ_a, X and $\hat{\lambda}_a$, $\hat{X}$ by each other [Bando, et al. (1993), Gulov, Skalozub (2000)]

If underlying theory is not specified, the set of $\hat{\lambda}_a$, $\hat{X}$ is unknown. The low energy theory consists of the SM plus the effective Lagrangian generated by the interactions of light particles with virtual heavy particle states. The low energy parameters $\lambda'_{l.}$ of these interactions are arbitrary numbers which must be constrained by experiments. By calculating the RG operator D and the scattering amplitudes of light particles in this "external field" in a chosen order of loop expansion, it is possible to obtain the model-independent correlations between $\lambda'_{l.}$. *These are just the RE relations*.

3 RG relations due to Z'

Let us derive the correlations between $\tilde{Y}_{\phi_{i,1}}$, $\tilde{Y}_{\phi_{i,2}}, \tilde{Y}_{L,f}, \tilde{Y}_{R,f}$ appearing due to renormalizability of the underlying theory containing Z'.

In our case, the RG invariance of the vertex leads to the equation

$$D\left(\bar{f}\Gamma_{fZ'}f\frac{1}{m'_Z}\right)=0, \quad D=\frac{d}{d\log\mu}=\frac{\partial}{\partial\log\mu}+\sum_a\beta_a\frac{\partial}{\partial\lambda_a}-\sum_X\gamma_X\frac{\partial}{\partial\log X}, \quad (11)$$

where

$$\beta_a=\frac{d\lambda}{d\log\mu}, \quad \gamma_X=-\frac{d\log X}{d\log\mu} \quad (12)$$

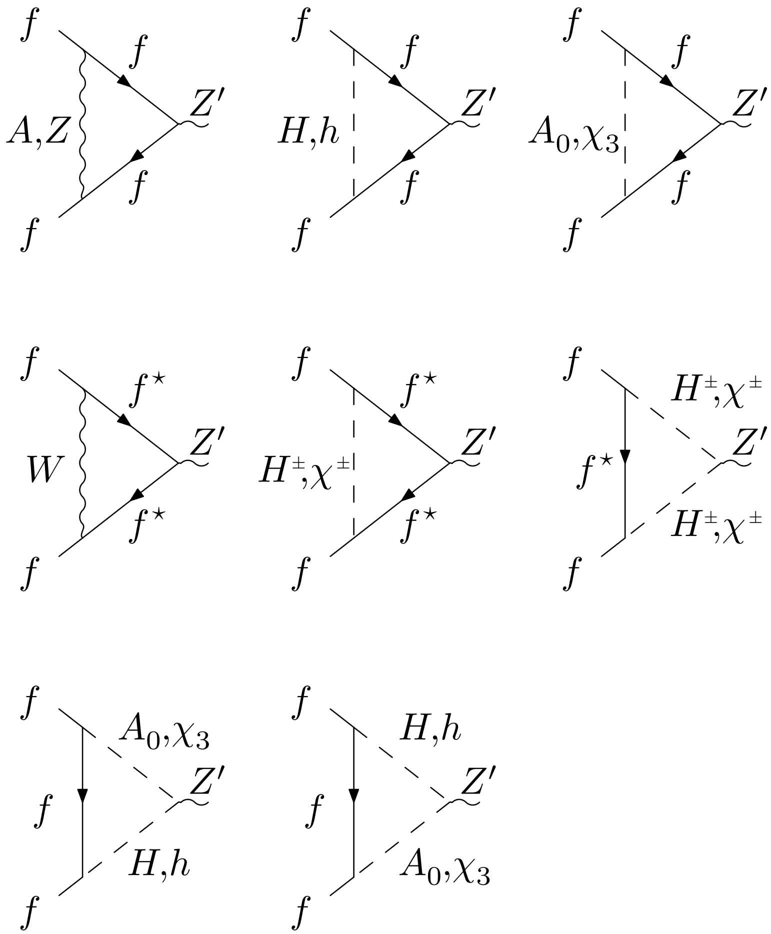
are computed with taking into account the loops of light particles.

In what follows, we derive the RG relations following from the one-loop consideration. In accordance with the previous sections, the one-loop RG equation for the vertex function reads

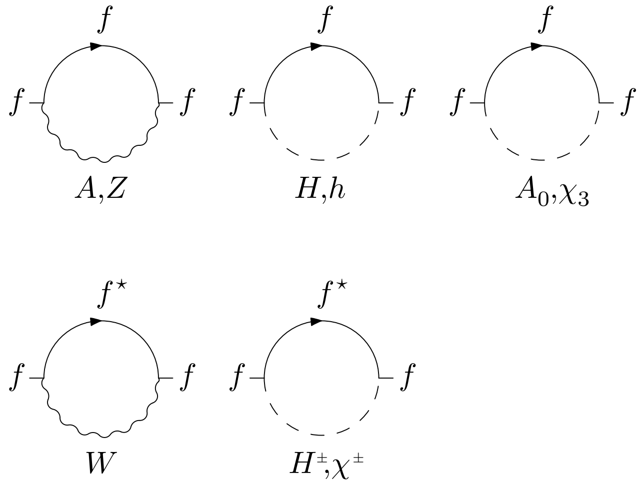
$$\bar{f}\frac{\partial\Gamma^{(1)}_{fZ'}}{\partial\log\mu}f\frac{1}{m_{Z'}}+D^{(1)}(\bar{f}\Gamma^{(0)}_{fZ'}f\frac{1}{m_{Z'}})=0, \quad (13)$$

$\Gamma^{(0)}_{fZ'}$, $\Gamma^{(1)}_{fZ'}$ **are the tree-level and one-loop contributions to the ffZ' vertex.** $D^{(1)}=\sum_a\beta^{(1)}_a\frac{\partial}{\partial\lambda_a}-\sum_X\gamma^{(1)}_X\frac{\partial}{\partial\log X}$ **is the one-loop level part of the RG operator.**

To calculate these functions, only the divergent parts of the one-loop vertices are to be calculated:



The fermion anomalous dimensions $\gamma_X^{(1)}$ are calculated by using the diagrams:



Suppose the same charge $\tilde{g}$ for the left-handed and right-handed fermions (usual requirement for Z' boson). Then, Eq.(13) leads to algebraic equations for the parameters $\tilde{Y}_{\phi_{i,1}}$, $\tilde{Y}_{\phi_{i,2}}, \tilde{Y}_{L,f}$, and $\tilde{Y}_{R,f}$ which have two sets of solutions [Gulov, Skalozub (2000)]:

$$\begin{aligned} \tilde{Y}_{\phi_{2,1}} &= \tilde{Y}_{\phi_{1,1}} = -\tilde{Y}_{\phi,2} \equiv -\tilde{Y}_{\phi}, \\ \tilde{Y}_{L,f} &+ \tilde{Y}_{L,f^*} = 0, \ \tilde{Y}_{R,f} = 0, \qquad \textbf{(Chiral } Z'\textbf{)} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \tilde{Y}_{\phi_{1,1}} &= \tilde{Y}_{\phi_{2,1}} = \tilde{Y}_{\phi,2} \equiv \tilde{Y}_{\phi}, \\ \tilde{Y}_{L,f} &= \tilde{Y}_{L,f^*}, \quad \tilde{Y}_{R,f} = \tilde{Y}_{L,f} + 2T_f^3\ \tilde{Y}_{\phi}. \qquad \textbf{(Abelian } Z'\textbf{)} \end{aligned} \tag{15}$$

Here f and f^* are the partners of the $SU(2)_L$ fermion doublet ($l^* = \nu_l, \nu^* = l, q_u^* = q_d$ and $q_d^* = q_u$), T_f^3 is the third component of weak isospin.

The first of these relations describes the Z' boson analogous to the third component of the $SU(2)_L$ gauge field. The couplings to the right-handed singlet are absent.

The second relation corresponds to the Abelian Z'. In this case the SM Lagrangian appears to be invariant with respect to the $\tilde{U}(1)$ group associated with the Z'. The last relation in Eq.(15) ensures the $L_{Yuk.}$ Eq.(3) is to be invariant with respect to the $\tilde{U}(1)$ transformations.

Introducing the Z' couplings to the vector and axial-vector fermion currents,

$$v^f_{Z'} = \frac{\tilde{Y}_{L,f} + \tilde{Y}_{R,f}}{2}, \qquad a^f_{Z'} = \frac{\tilde{Y}_{R,f} - \tilde{Y}_{L,f}}{2}, \tag{16}$$

the last line in Eq.(15) yields

$$v^f_{Z'} - a^f_{Z'} = v^{f^*}_{Z'} - a^{f^*}_{Z'}, \qquad a^f_{Z'} = T^3_f \tilde{Y}_\phi. \tag{17}$$

The couplings of the Abelian Z' to the axial-vector fermion current have a universal absolute value proportional to the Z' coupling to the scalar doublet.

These relations are model independent. In particular, they hold in all the known models containing the Abelian Z'.

f	χ-ψ		**LR**	
	$a_f/\tilde{g}$	$v_f/\tilde{g}$	$a_f/\tilde{g}$	$v_f/\tilde{g}$
ν	$-3\frac{\cos\beta}{\sqrt{40}} - \frac{\sin\beta}{\sqrt{24}}$	$3\frac{\cos\beta}{\sqrt{40}} + \frac{\sin\beta}{\sqrt{24}}$	$-\frac{1}{2\alpha}$	$\frac{1}{2\alpha}$
e	$-\frac{\cos\beta}{\sqrt{10}} - \frac{\sin\beta}{\sqrt{6}}$	$2\frac{\cos\beta}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$\frac{1}{\alpha} - \frac{\alpha}{2}$
q_u	$\frac{\cos\beta}{\sqrt{10}} - \frac{\sin\beta}{\sqrt{6}}$	$\mathbf{0}$	$\frac{\alpha}{2}$	$-\frac{1}{3\alpha} + \frac{\alpha}{2}$
q_d	$-\frac{\cos\beta}{\sqrt{10}} - \frac{\sin\beta}{\sqrt{6}}$	$-2\frac{\cos\beta}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$-\frac{1}{3\alpha} - \frac{\alpha}{2}$

The most discussed models are derived from the E_6 group:

$$\mathrm{E}_6 \to \mathrm{SO}(10)\times\mathrm{U}(1)_\psi, \quad \mathrm{SO}(10)\to\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}. \qquad \textbf{(LR)}$$

$$\mathrm{E}_6 \to \mathrm{SO}(10)\times\mathrm{U}(1)_\psi \to \mathrm{SU}(5)\times\mathrm{U}(1)_\chi\times\mathrm{U}(1)_\psi, \qquad (\chi-\psi, \textbf{ two neutral bosons})$$

$$Z' = \chi\cos\beta + \psi\sin\beta.$$

If we suppose only one Z' boson at low energies, the ψ boson should be much heavier than the χ field. In this case, ψ is decoupled and $\beta \to 0$.

In E_6 theories there are no Yukawa terms responsible for generation of the Dirac masses of neutrinos. The Yukawa couplings of neutrinos vanish in the RG equation, and a_ν are not restricted by the RG relations.

Z' coupling parametrization: results

The considered Lagrangian leads to the (usual) interactions between the fermions and the Z and Z' mass eigenstates:

$$\begin{aligned}\mathcal{L}_{Z\bar{f}f} &= \frac{1}{2} i Z_\mu \bar{f}\gamma^\mu \left[(v^{\rm SM}_{fZ} + \gamma^5 a^{\rm SM}_{fZ})\cos\theta_0 + \right. \\ &\quad \left. +(v_f + \gamma^5 a_f)\sin\theta_0\right] f, \\ \mathcal{L}_{Z'\bar{f}f} &= \frac{1}{2} i Z'_\mu \bar{f}\gamma^\mu \left[(v_f + \gamma^5 a_f)\cos\theta_0 - \right. \\ &\quad \left. -(v^{\rm SM}_{fZ} + \gamma^5 a^{\rm SM}_{fZ})\sin\theta_0\right] f, \end{aligned} \qquad (18)$$

where f is an arbitrary SM fermion state; $v^{\rm SM}_{fZ}$, $a^{\rm SM}_{fZ}$ are the SM couplings of the Z-boson; θ_0 is the Z-Z' mixing angle.

Since the Z' couplings enter the cross-section together with the inverse Z' mass, it is convenient to introduce the dimensionless couplings, which can be constrained by experiments:

$$\bar{a}_f = \frac{m_Z}{\sqrt{4\pi} m_{Z'}} a_f, \quad \bar{v}_f = \frac{m_Z}{\sqrt{4\pi} m_{Z'}} v_f,$$

In case of the Abelian Z'-boson, the Z–Z' mixing angle θ_0 is determined by the vector-scalar coupling $\tilde{Y}_\phi$ as follows

$$\theta_0 = \frac{\tilde{g}\sin\theta_W\cos\theta_W}{\sqrt{4\pi\alpha_{\mathrm{em}}}}\frac{m_Z^2}{m_{Z'}^2}\tilde{Y}_\phi + O\left(\frac{m_Z^4}{m_{Z'}^4}\right), \qquad (19)$$

where θ_W is the SM Weinberg angle, and α_{em} is the electromagnetic fine structure constant.

Although the mixing angle is a small quantity of order $m_{Z'}^{-2}$, it contributes to the Z-boson exchange amplitude and cannot be neglected at the LEP energies.

The axial-vector coupling determines also the coupling to the scalar doublet and, consequently, the mixing angle. As a result, the number of independent couplings is significantly reduced. Note also that the absolute value of the axial-vector Z' coupling is flavor-independent:

$$\bar{a} = \bar{a}_{e,\,\mu,\,\tau,\,d-\text{type quarks}} = -\bar{a}_{\nu,\,u-\text{type quarks}} \sim \theta_0/m_{Z'}.$$

The RG relations give a possibility:

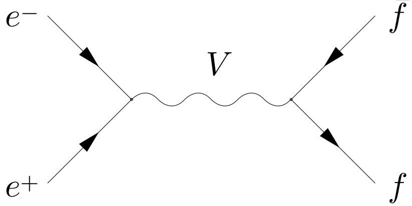
1) reduce the number of fitted parameters (v_f, a);

2) taking into account the kinematics of the processes introduce observables which uniquely pick out the Z' signals.

4 Annihilation processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$

We consider the processes $e^+e^- \to l^+l^-$ $(l = \mu, \tau)$ with the non-polarized initial- and final-state fermions. In order to introduce the observable which selects the signal of the Abelian Z' boson we compute the differential cross-sections of the processes up to the one-loop level.

The lower-order diagrams describe the neutral vector boson exchange in the s-channel ($e^+e^- \to V^* \to l^+l^-$, $V = A, Z, Z'$).

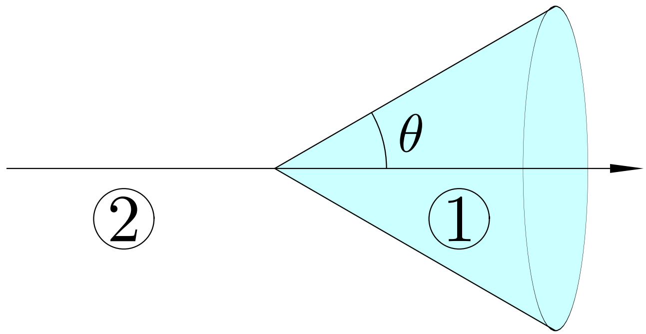


In the lower order in $m_{Z'}^{-2}$ the Z' contributions to the differential cross-section of the process $e^+e^- \to l^+l^-$ are expressed in terms of four-fermion contact couplings, only.

If one takes into consideration the higher-order corrections in $m_{Z'}^{-2}$, it becomes possible to estimate separately the Z'-induced contact couplings and the Z' mass [Hewett, Rizzo (1989)]. In the present analysis we keep the terms of order $O(m_{Z'}^{-4})$ to fit both of these parameters.

We introduce the observable (generalized forward-backward cross-section)

$$\sigma_l(z) = \int_z^1 \frac{d\sigma_l}{d\cos\theta} d\cos\theta - \int_{-1}^z \frac{d\sigma_l}{d\cos\theta} d\cos\theta,$$



$$\Delta\sigma_l(z^*) \sim -\bar{a}^2, \qquad z^* = 0.38 \text{ at } \sqrt{s} = 200\,\mathrm{GeV}.$$

We present the final result of the analysis carried out. The fits were performed which assumed several data sets, including the $\mu\mu$, $\tau\tau$, and the complete $\mu\mu$ and $\tau\tau$ data, respectively. The results are presented in Table.

The dimensionless axial-vector contact coupling $\bar{a}^2$ with the 68% confidence-level uncertainty, the probability of the Z' signal, P, and the value of $m_Z^2/m_{Z'}^2$ as a result of the fit of the observable recalculated from the total cross-sections and the forward-backward asymmetries.

Data set	$\bar{a}^2$	P	$m_Z^2/m_{Z'}^2$
$\mu\mu$	$0.0000366^{+0.0000489}_{-0.0000486}$	**0.77**	0.009 ± 0.278
$\tau\tau$	$-0.0000266^{+0.0000643}_{-0.0000639}$	**0.34**	-0.001 ± 0.501
$\mu\mu$ **and** $\tau\tau$	$0.0000133^{+0.0000389}_{-0.0000387}$	**0.63**	0.017 ± 0.609

As it is seen, the more precise $\mu\mu$ data demonstrate the hint of about 1σ level. It corresponds to the Abelian Z'-boson with the mass of order 1.2–1.5TeV if one assumes the value of $\tilde{\alpha} = \tilde{g}^2/4\pi$ to be in the interval 0.01–0.02.

5 Bhabha process $e^+e^- \to e^+e^-$

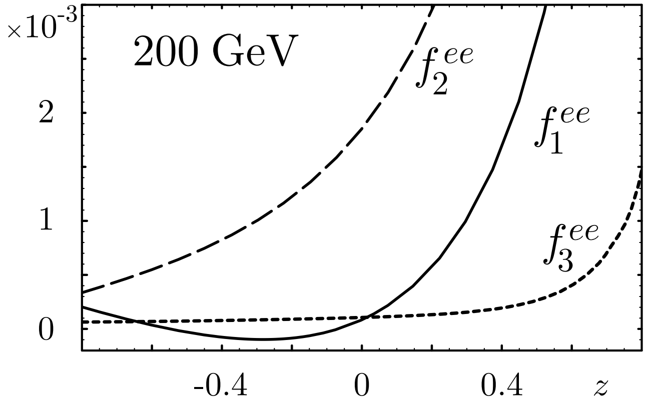
Differential cross-section

In our analysis, as the SM values of the cross-sections we use the quantities calculated by the LEP2 collaborations. The deviation from the SM is computed in the improved Born approximation.

The deviation from the SM of the differential cross-section for the process $e^+e^- \to e^+e^-$ can be expressed through quadratic combinations of couplings a, v_e,

$$\frac{d\sigma}{dz} - \frac{d\sigma^{\mathrm{SM}}}{dz} = f_1^{ee}(z)\frac{a^2}{m_{Z'}^2} + f_2^{ee}(z)\frac{v_e^2}{m_{Z'}^2} + f_3^{ee}(z)\frac{av_e}{m_{Z'}^2}, \quad (20)$$

where the factors are known functions of the center-of-mass energy and the cosine of the electron scattering angle z plotted in Fig.



It is convenient to introduce the dimensionless couplings

$$\bar{a}_f = \frac{m_Z}{\sqrt{4\pi} m_{Z'}} a_f, \quad \bar{v}_f = \frac{m_Z}{\sqrt{4\pi} m_{Z'}} v_f, \tag{21}$$

which can be constrained by experiments.

The cross-sections in Eq. (20) account for the relations (15) through the functions $f_1(z)$, $f_3(z)$.

One-parameter fit

The factor $f_2^{ee}(z) = \mathcal{F}_v(\sqrt{s}, z)$ is positive monotonic function of z. Such a property allows one to choose $\mathcal{F}_v(\sqrt{s}, z)$ as a normalization factor for the differential cross section. Then the normalized deviation reads

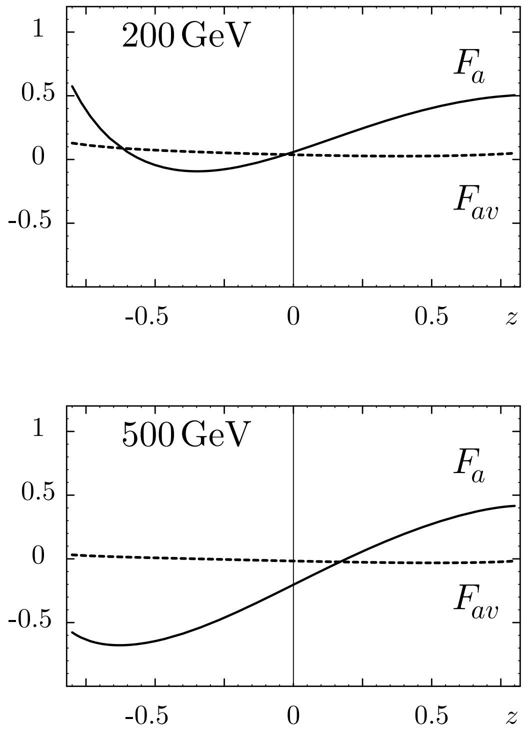
$$\begin{aligned} \frac{d\tilde{\sigma}}{dz} &= \mathcal{F}_v^{-1}(\sqrt{s}, z) \Delta \frac{d\sigma}{dz} = \\ &= \bar{v}^2 + F_a(\sqrt{s}, z)\bar{a}^2 + F_{av}(\sqrt{s}, z)\bar{a}\bar{v} + \ldots, \end{aligned} \tag{22}$$

and the normalized factors are finite at $z \to 1$. Each of them in a special way influences the differential cross-section.

1. The factor at $\bar{v}^2$ is just the unity. Hence, the four-fermion contact coupling between vector currents, $\bar{v}^2$, determines the level of the deviation from the SM value.

2. The factor at $\bar{a}^2$ depends on the scattering angle in a non-trivial way. It allows to recognize the Abelian Z' boson, if the experimental accuracy is sufficient.

3. The factor at $\bar{a}\bar{v}$ results in small corrections.



$F_a(\sqrt{s}, z)$: solid, $F_{av}(\sqrt{s}, z)$: dashed, $\sqrt{s} = 200$, 500 GeV.

Thus, effectively, the obtained normalized differential cross-section is a two-parametric function.

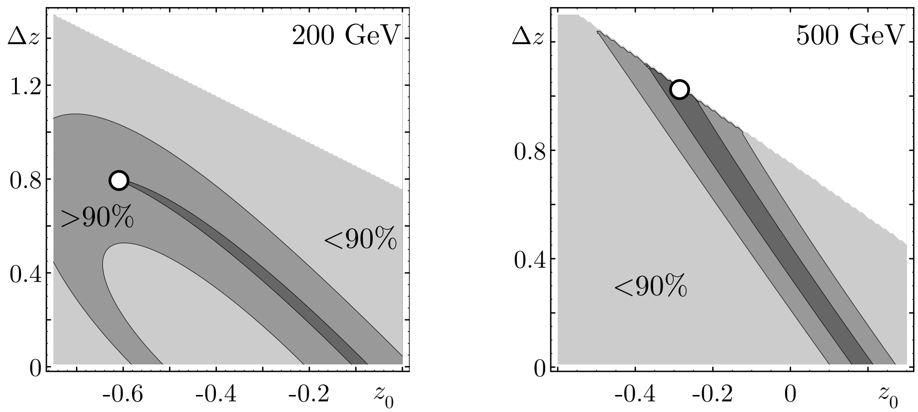
Observable to pick out $\bar{v}^2$. After normalization the factor at $\bar{v}^2$ is 1. The factor at $\bar{a}^2$ is a sign-varying function which is small over the backward scattering angles for $\sqrt{s} \sim 200$ GeV. So, to measure $\bar{v}^2$ the normalized deviation of the differential cross-section has to be integrated over the backward angles:

$$\sigma_V = \int_{z_0}^{z_0+\Delta z} (d\tilde{\sigma}/dz)dz,$$

where at each energy the most effective interval $[z_0, z_0 + \Delta z]$ is determined by the following requirements:

1. **The relative contribution of the coupling $\bar{v}^2$ is maximal.** Equivalently, the contribution of the factor at $\bar{a}^2$ is suppressed.

2. **The length Δz of the interval is maximal.** This condition ensures that the largest number of bins is taken into consideration.



$-0.6 < z < 0.2,$

$\sqrt{s} = 200$ **GeV**

The values of the Z' coupling to the electron vector current together with their 1σ uncertainties are [Gulov, Skalozub (Phys. Rev. D, 2007)]:

$$\begin{aligned} \text{ALEPH}: \quad \bar{v}_e^2 &= -0.11 \pm 6.53 \times 10^{-4} \\ \text{DELPHI}: \quad \bar{v}_e^2 &= 1.60 \pm 1.46 \times 10^{-4} \\ \text{L3}: \quad \bar{v}_e^2 &= 5.42 \pm 3.72 \times 10^{-4} \\ \text{OPAL}: \quad \bar{v}_e^2 &= 2.42 \pm 1.27 \times 10^{-4} \\ \text{Combined}: \quad \bar{v}_e^2 &= 2.24 \pm 0.92 \times 10^{-4}. \end{aligned}$$

As one can see, the most precise data of DELPHI and OPAL collaborations are resulted in the Abelian Z' hints at 1 and 2σ CL, correspondingly. The combined value shows the 2σ hint, which corresponds to $0.006 \leq |\bar{v}_e| \leq 0.020$.

Many-parameter fits

Now we fulfill the many parametric fits accounting for the total amount of the LEP2 experiment data.

As the basic observable to fit the LEP2 experiment data on the Bhabha process we propose the final differential cross-sections measured by the ALEPH (130-183 GeV), DELPHI (189-207 GeV), L3 (183-189 GeV), and OPAL (130-207 GeV) collaborations, $\left.\frac{d\sigma^{\text{Bhabha}}}{dz}\right|_{z=z_i,\sqrt{s}=\sqrt{s_i}}$**, where i runs over the bins at various center-of-mass energies $\sqrt{s}$ (299 bins).**

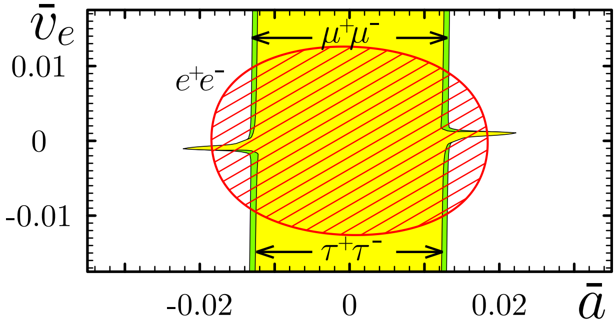
As the observables for $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ processes, we consider the total cross-section $\boxed{\sigma_T^{\ell^+\ell^-}|_{\sqrt{s}=\sqrt{s_i}}}$ and the forward-backward asymmetry $\boxed{A_{FB}^{\ell^+\ell^-}|_{\sqrt{s}=\sqrt{s_i}}}$ where i runs over 12 center-of-mass energies $\sqrt{s}$ from 130 to 207 GeV. We consider the combined LEP2 data [Electroweak Working Group (2006)] for these observables (24 data entries for each process). These data are more precise as the corresponding differential cross-sections.

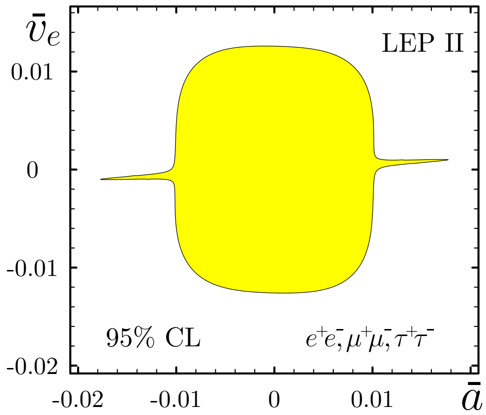
The data are analyzed by means of the χ^2 fit. Denoting the observables by σ_i, one can construct the χ^2-function,

$$\boxed{\chi^2(\bar{a}, \bar{v}_e, \bar{v}_\mu, \bar{v}_\tau) = \sum_i \left[\frac{\sigma_i^{\rm ex} - \sigma_i^{\rm th,SM} - \sigma_i^{{\rm th},Z'}(\bar{a}, \bar{v}_e, \bar{v}_\mu, \bar{v}_\tau)}{\delta\sigma_i} \right]^2, \qquad (23)}$$

where $\sigma^{\rm ex}$ and $\delta\sigma$ are the experimental values and the uncertainties of the observables, $\sigma^{\rm th,SM}$ are the SM values of the observables, and $\sigma^{{\rm th},Z'}$ are the deviations from the SM value due to the Z' boson. The sum in Eq. (23) refers to either the data for one specific process or the combined data for several processes.

The 95% CL areas in the $(\bar{a}, \bar{v}_e)$ plane for the separate processes are plotted in Fig. The Bhabha process constrains both the axial-vector and vector couplings. As for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes, the axial-vector coupling is significantly constrained, only.





The projection of the 95% CL area onto the $(\bar{a}, \bar{v}_e)$ plane for the combination of the Bhabha, $e^+e^- \to \mu^+\mu^-$, and $e^+e^- \to \tau^+\tau^-$ processes.

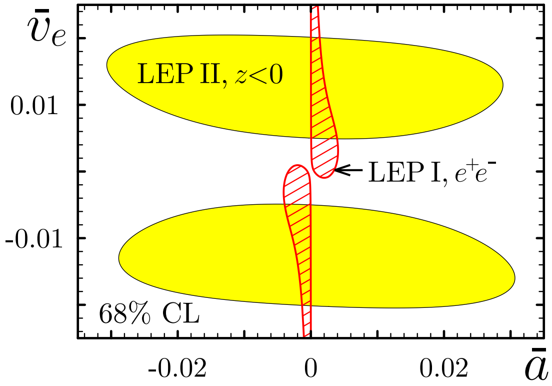
Let us compare the obtained results with the one-parameter fits.

As one can see, the most precise data of DELPHI and OPAL collaborations are resulted in the Abelian Z' hints at one and two standard deviation level, correspondingly. The combined value shows the 2σ hint, which corresponds to $0.006 \leq |\bar{v}_e| \leq 0.020$.

Our one-parameter observable accounts mainly for the backward bins. This is in accordance with the kinematic features of the process: the backward bins depend mainly on the vector coupling $\bar{v}_e^2$, whereas the contributions of other couplings are kinematically suppressed (see Fig. after Eq. 21).

We perform the many-parameter fit with the 113 backward bins ($z \leq 0$), only. The χ^2 minimum, $\chi^2_{\rm min} = 93.0$, is found in the non-zero point $|\bar{a}| = 0.0005$, $\bar{v}_e = 0.015$. This value of the Z' coupling $\bar{v}_e$ is in an excellent agreement with the mean value obtained in the one-parameter fit.

The 68% CA in the $(\bar{a}, \bar{v}_e)$ plane is plotted in Fig. The zero point $\bar{a} = \bar{v}_e = 0$ (the absence of the Z' boson) corresponds to $\chi^2 = 97.7$. It is covered by the CA with 1.3σ CL. Thus, the backward bins show the 1.3σ hint of the Abelian Z' boson in the many-parameter fit.



The summary of the fits of the LEP data for the dimensionless contact couplings.

Data	$\bar{v}_e^2$	$\bar{a}^2$
LEP1		
e^-e^+, **68% CL**	**-**	$(1.25 \pm 1.25) \times 10^{-5}$
LEP2, one-parameter fits		
e^-e^+, **68% CL**	$(2.24 \pm 0.92) \times 10^{-4}$	**-**
$\mu\mu$, **68% CL**	**-**	$(3.66^{+4.89}_{-4.86}) \times 10^{-5}$
$\mu\mu$,$\tau\tau$, **68% CL**	**-**	$(1.33^{+3.89}_{-3.87}) \times 10^{-5}$
LEP2, many-parameter fits		
$e^-e^+, \mu\mu, \tau\tau$, **95% CL**	$\leq 1.69 \times 10^{-4}$	$\leq 3.61 \times 10^{-4}$
e^-e^+ **backward, 68% CL**	$(2.25^{+1.79}_{-2.07}) \times 10^{-4}$	$\leq 9.49 \times 10^{-4}$

Let us present the results of fits of the Z' parameters in terms of the standard notations [Leike (1998), Langacker (2008)]. The Lagrangian reads

$$\begin{aligned}\mathcal{L}_{Z\bar{f}f} &= \frac{1}{2} Z_\mu \bar{f} \gamma^\mu \left[(v_f^{\rm SM} + \Delta_f^V) - \gamma^5 (a_f^{\rm SM} + \Delta_f^A) \right] f, \\ \mathcal{L}_{Z'\bar{f}f} &= \frac{1}{2} Z'_\mu \bar{f} \gamma^\mu \left[(v'_f - \gamma^5 a'_f) \right] f, \end{aligned} \tag{24}$$

with the SM values of the Z couplings

$$v_f^{\rm SM} = \frac{e \left(T_{3f} - 2 Q_f \sin^2 \theta_W \right)}{\sin \theta_W \cos \theta_W}, \qquad a_f^{\rm SM} = \frac{e\, T_{3f}}{\sin \theta_W \cos \theta_W},$$

where e is the positron charge, Q_f is the fermion charge in the units of e, $T_{3f} = 1/2$ for the neutrinos and u-type quarks, and $T_{3f} = -1/2$ for the charged leptons and d-type quarks.

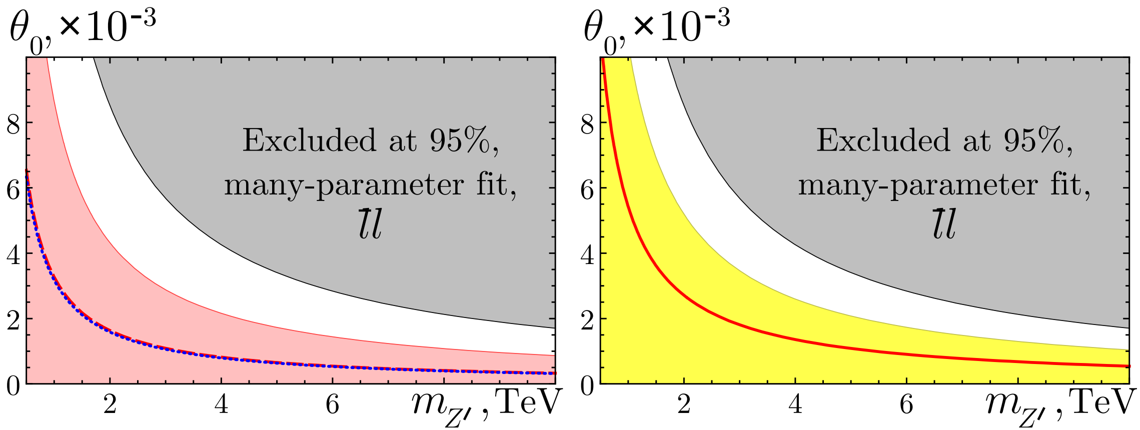
The summary of the fits of the LEP data for the maximum likelihood values of the Z' couplings to the SM fermions and of the $Z-Z'$ mixing angle θ_0.

$M = \frac{m_{Z'}}{1\,\mathrm{TeV}}$ denotes the unknown value of the Z' mass in TeV units.

Data	$\lvert\theta_0\rvert$, $\times 10^{-3}$	$\lvert v'_e\rvert$, $\times 10^{-1}$	$\lvert a'_f\rvert$, $\times 10^{-1}$	Δ^A_e, $\times 10^{-3}$
LEP1				
e^-e^+	$3.17 M^{-1}$	-	$1.38 M$	**0.437**
LEP2, one-parameter fits				
e^-e^+	-	$5.83 M$	-	-
$\mu^-\mu^+$	$5.42 M^{-1}$	-	$2.36 M$	**1.278**
$\mu^-\mu^+, \tau^-\tau^+$	$3.27 M^{-1}$	-	$1.42 M$	**0.464**
LEP2, many-parameter fits				
e^-e^+, $z < 0$	-	$5.84 M$	-	-

The summary of the fits of the LEP data for the confidence intervals for the Z' couplings to the SM fermions and for the $Z-Z'$ mixing angle θ_0. $M=\frac{m_{Z'}}{1\,\mathrm{TeV}}$ denotes the unknown value of the Z' mass in TeV units.

Data	CL	$\lvert\theta_0\rvert$, $\times 10^{-3}$	$\lvert v'_e\rvert$, $\times 10^{-1}$	$\lvert a'_f\rvert$, $\times 10^{-1}$	Δ^A_e, $\times 10^{-3}$
LEP1					
e^-e^+	**68%**	$(0, 4.48)M^{-1}$	-	$(0, 1.95)M$	$(0, 0.873)$
LEP2, one-parameter fits					
e^-e^+	**95%**	-	$(2.46, 7.87)M$	-	-
$\mu^-\mu^+$	**95%**	$(0, 10.39)M^{-1}$	-	$(0, 4.52)M$	$(0, 4.694)$
$\mu^-\mu^+$, $\tau^-\tau^+$	**95%**	$(0, 8.64)M^{-1}$	-	$(0, 3.75)M$	$(0, 3.244)$
LEP2, many-parameter fits					
e^-e^+, $\mu^-\mu^+$, $\tau^-\tau^+$	**95%**	$(0, 17.03)M^{-1}$	$(0, 5.06)M$	$(0, 7.40)M$	$(0, 12.607)$
e^-e^+, $z<0$	**68%**	$(0, 27.61)M^{-1}$	$(1.68, 7.83)M$	$(0, 12.00)M$	$(0, 33.1288)$

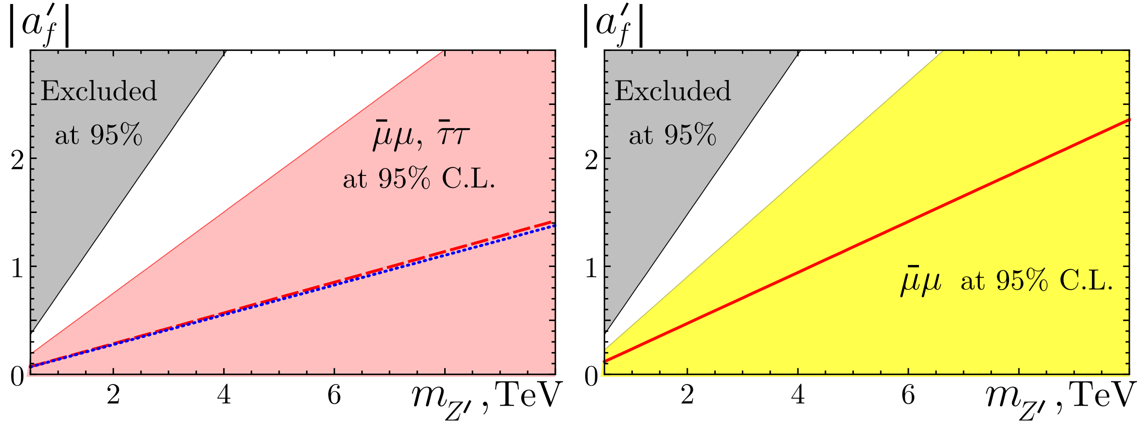


GRAY: the values excluded at 95% CL by the many-parameter fit of $e^+e^- \to l^+l^-$**.**

PINK: fits of the one-parameter observables for $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ **(with the maximum likelihood value as the DASHED RED line)**

YELLOW: fits of the one-parameter observables for $e^+e^- \to \mu^+\mu^-$ **(with the maximum likelihood value as the SOLID RED line)**

DOTTED BLUE: the maximum likelihood values from the LEP1 data.

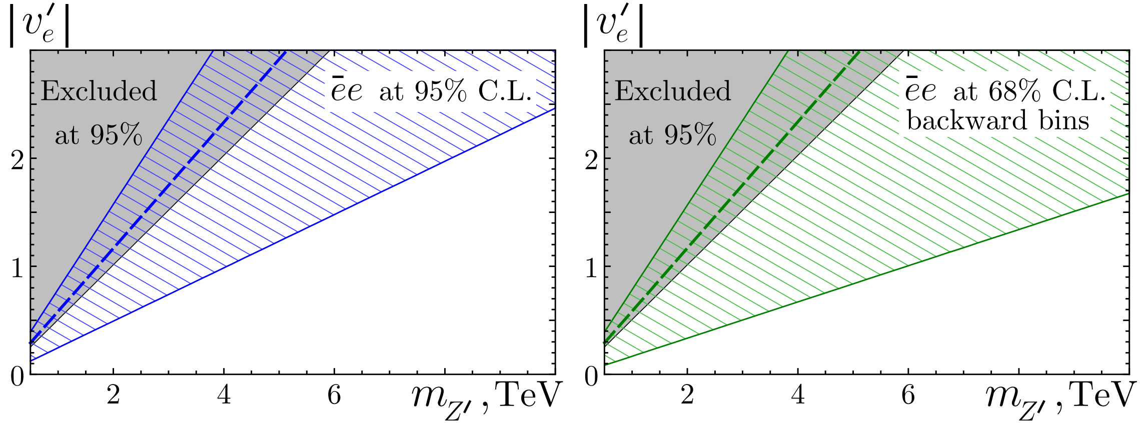


GRAY: the values excluded at 95% CL by the many-parameter fit of $e^+e^- \to l^+l^-$**.**

PINK: fits of the one-parameter observables for $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ **(with the maximum likelihood value as the DASHED RED line)**

YELLOW: fits of the one-parameter observables for $e^+e^- \to \mu^+\mu^-$ **(with the maximum likelihood value as the SOLID RED line)**

DOTTED BLUE: the maximum likelihood values from the LEP1 data.



GRAY: the values excluded at 95% CL by the many-parameter fit of $e^+e^- \to l^+l^-$.

BLUE: fits of the one-parameter observables for $e^+e^- \to e^+e^-$ (with the maximum likelihood value as the DASHED LINE).

GREEN: The maximum likelihood values (DASHED LINE) and the 1σ CL area for the many-parameter fit of backward bins of $e^+e^- \to e^+e^-$.

Conclusion

LEP collaborations have obtained model dependent low bounds on the Z' mass. It varies from 400 to 800 GeV at 95 per cent CL dependently on the Z' model. A possibility to select Z' signal in specific scattering processes was not considered.

Our analysis of the leptonic processes based on the same data set and the same SM values of the cross-sections showed that the existence of Z' boson with the mass of order 1 - 1.2 TeV is not excluded at the 1 - 2 σ CL, being compatible with the LEP2 reports.

The estimated Z' parameters $\bar{v}_e^2 = 2.24 \pm 0.92 \times 10^{-4}$ and $\bar{a}^2 = 1.3 \pm 3.89 \times 10^{-5}$ at 68 per sent CL derived in different methods are in good agreement with each other.

In our analysis, the RG relations play a crucial role in treating experimental data. They served to reduce the number of unknown parameters and extract a maximal information about the Z' from the experimental data set. If the RG relations are not taken into account no hints of Z' will be found.

We believe that the RG relations will be also important in searches for the Z' at LHC.