Sigma models for Lorentz group and superluminal propagation in 2d

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work in progress

Is Lorentz symmetry exact?

- Explore departures from LS
- Explicit violation of LS incompatible with general relativity
- Consider spontaneous LS breaking:
  the action is Lorentz invariant (and generally covariant),
  but the vacuum is not

Sigma-model

$$SO(3,1)/SO(3) \quad \Rightarrow \quad V_\mu V^\mu = M^2$$

preserve spatial isotropy

Einstein-aether

T. Jacobson, D. Mattingly (2001)

$$S = \int d^4x \Big[-\alpha_1 \partial_\mu V^\nu \partial^\mu V_\nu - \alpha_2 (\partial_\mu V^\mu)^2$$
$$-\alpha_3 \epsilon^{\mu\nu\lambda\rho} \partial_\mu V_\nu \partial_\lambda V_\rho - \alpha_4 V^\mu \partial_\mu V^\nu V^\lambda \partial_\lambda V_\nu$$
$$+\lambda (V_\mu V^\mu - M^2) \Big]$$

Requires UV completion: $\Lambda \leq M$

2d toy-models may be useful: better quantum properties

C. Eling, T. Jacobson (2006)

$$S = \int d^2x \Big(-\alpha_1 \partial_\mu V^\nu \partial^\mu V_\nu - \alpha_2 \partial_\mu V^\mu \partial_\nu V^\nu$$
$$- \alpha_3 \partial_\mu V^\mu \epsilon^{\nu\lambda} \partial_\nu V_\lambda + \lambda (V^\mu V_\mu - 1) \Big)$$

violates parity

$V_\mu V^\mu = 1$

Work in light-cone coordinates $x^{\pm} = \frac{1}{\sqrt{2}}(t \pm x)$

Introduce "rapidity" field: $V^{\pm} = \frac{1}{\sqrt{2}} \mathrm{e}^{\pm\psi}$

$$S = \int dx^+ dx^- \frac{1}{g^2} \left\{ \partial_+\psi \partial_-\psi + \frac{\beta_{(+)}}{2} (\partial_+\psi)^2 \mathrm{e}^{2\psi} + \frac{\beta_{(-)}}{2} (\partial_-\psi)^2 \mathrm{e}^{-2\psi} \right\}$$

$$g^2 = \frac{1}{2\alpha_1 + \alpha_2} \qquad \beta_{(\pm)} = \frac{-\alpha_2 \pm \alpha_3}{2\alpha_1 + \alpha_2}$$

- Lorentz symmetry is realized non-linearly:

$$\psi(x^+, x^-) \mapsto \psi(\mathrm{e}^{\gamma} x^+, \mathrm{e}^{-\gamma} x^-) + \gamma$$

- Renormalizable by power-counting

One-loop RG flow

g^2 does not run

$$\frac{d\beta_{(\pm)}}{d\log\Lambda} = -\frac{g^2\beta_{(\pm)}}{\pi\sqrt{1-\beta_{(+)}\beta_{(-)}}}$$

- In UV: $\beta_{(\pm)} \to 0$ ⇒ theory flows to the free Lorentz invariant limit
- In IR: three cases

$\beta_{(+)}, \beta_{(-)}$: ++, +0, +-

One-loop RG flow

$$\beta_{(+)} = \beta_{(-)} = \beta$$

$$\frac{d\beta}{d\log\Lambda} = -\frac{g^2\beta}{\pi\sqrt{1-\beta^2}}$$

Infrared pole $\beta = 1$ at finite scale ➡ strong coupling in IR

$$\beta_{(-)} = 0$$

No physical running: change of β is compensated by the shift of ψ

One-loop running

$$\beta_{(+)} = -\beta_{(-)} = \beta$$

$$\frac{d\beta}{d\log\Lambda} = -\frac{g^2\beta}{\pi\sqrt{1+\beta^2}}$$

In IR $\beta \to \infty$

$$S = \frac{1}{2\varkappa^2}\int dx_+ dx_- \left[(\partial_+\psi)^2 \mathrm{e}^{2\psi} - (\partial_-\psi)^2 \mathrm{e}^{-2\psi}\right]$$

$$\varkappa^2 = g^2/\beta$$

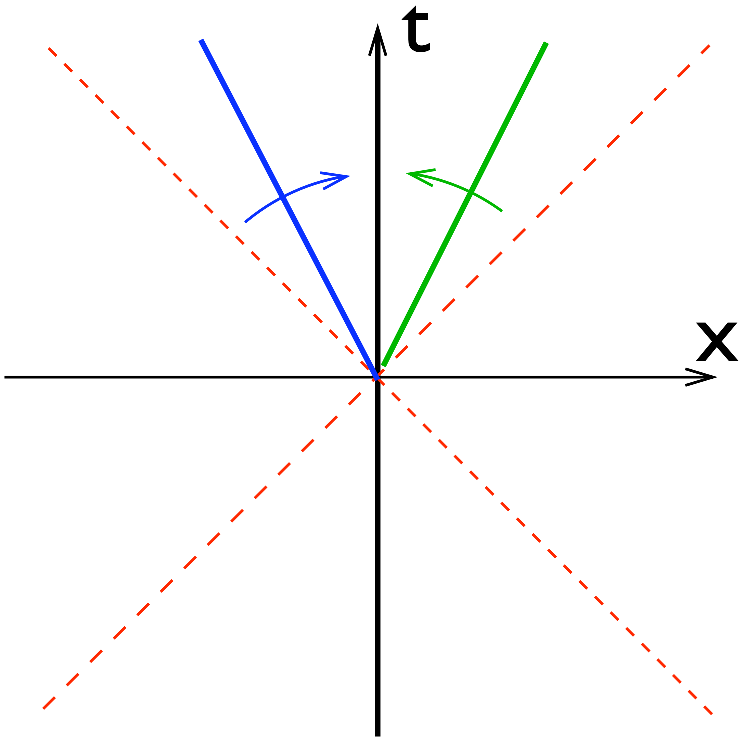
$$\frac{d\varkappa}{d\log\Lambda} = \frac{\varkappa^3}{2\pi}$$

theory flows to a weakly coupled point

Propagation of signals

$$\beta_{(+)} = \beta_{(-)} = \beta$$

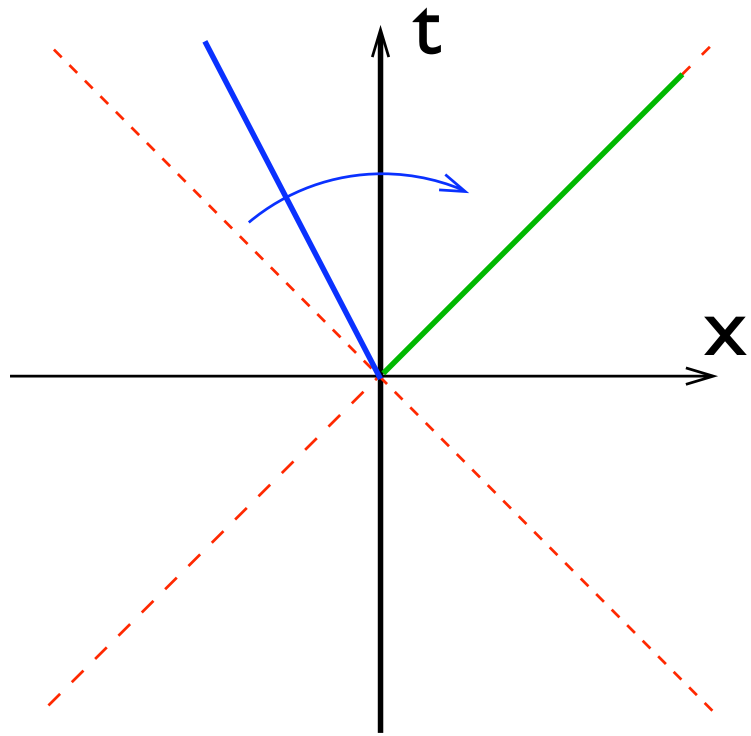
Fix background $\psi = 0$ $\Rightarrow$ $v^2 = \dfrac{1-\beta}{1+\beta}$



Propagation of signals

$$\beta_{(-)} = 0$$

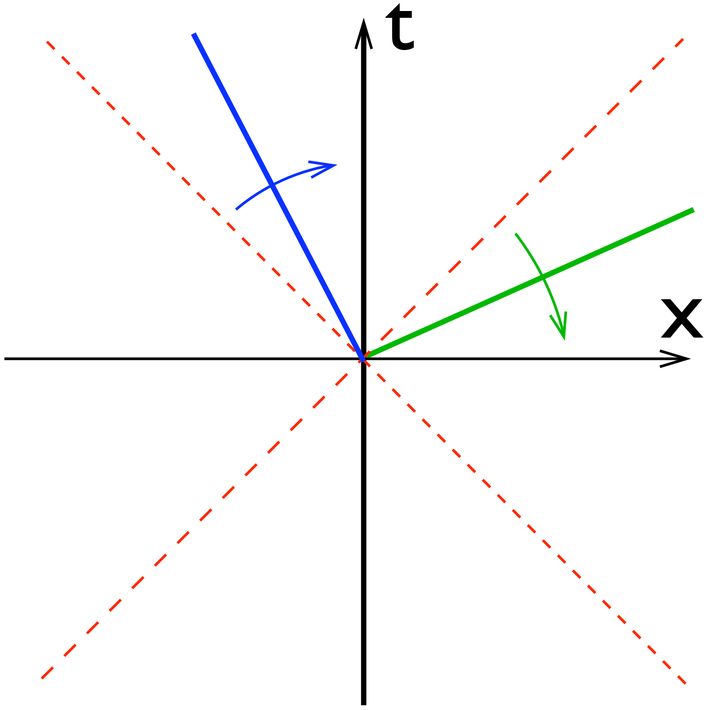
$$v_L = \frac{\beta - 2}{\beta + 2} \qquad\qquad v_R = 1$$



Propagation of signals

$$\beta_{(+)} = -\beta_{(-)} = \beta$$

$$v_L = -\beta + \sqrt{1+\beta^2} \qquad v_R = \beta + \sqrt{1+\beta^2}$$



The right-moving mode is superluminal

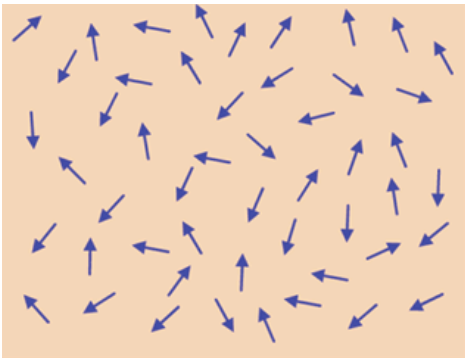
Restoration of Lorentz invariance

No spontaneous breaking of continuous symmetries in 2d

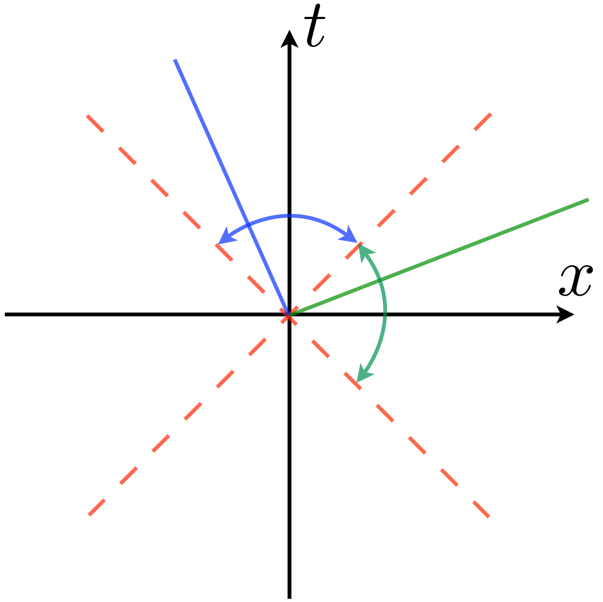
N.D. Mermin, H. Wagner (1966)
S. Coleman (1973)

Average over classical vacua.

Example: 2d ferromagnet

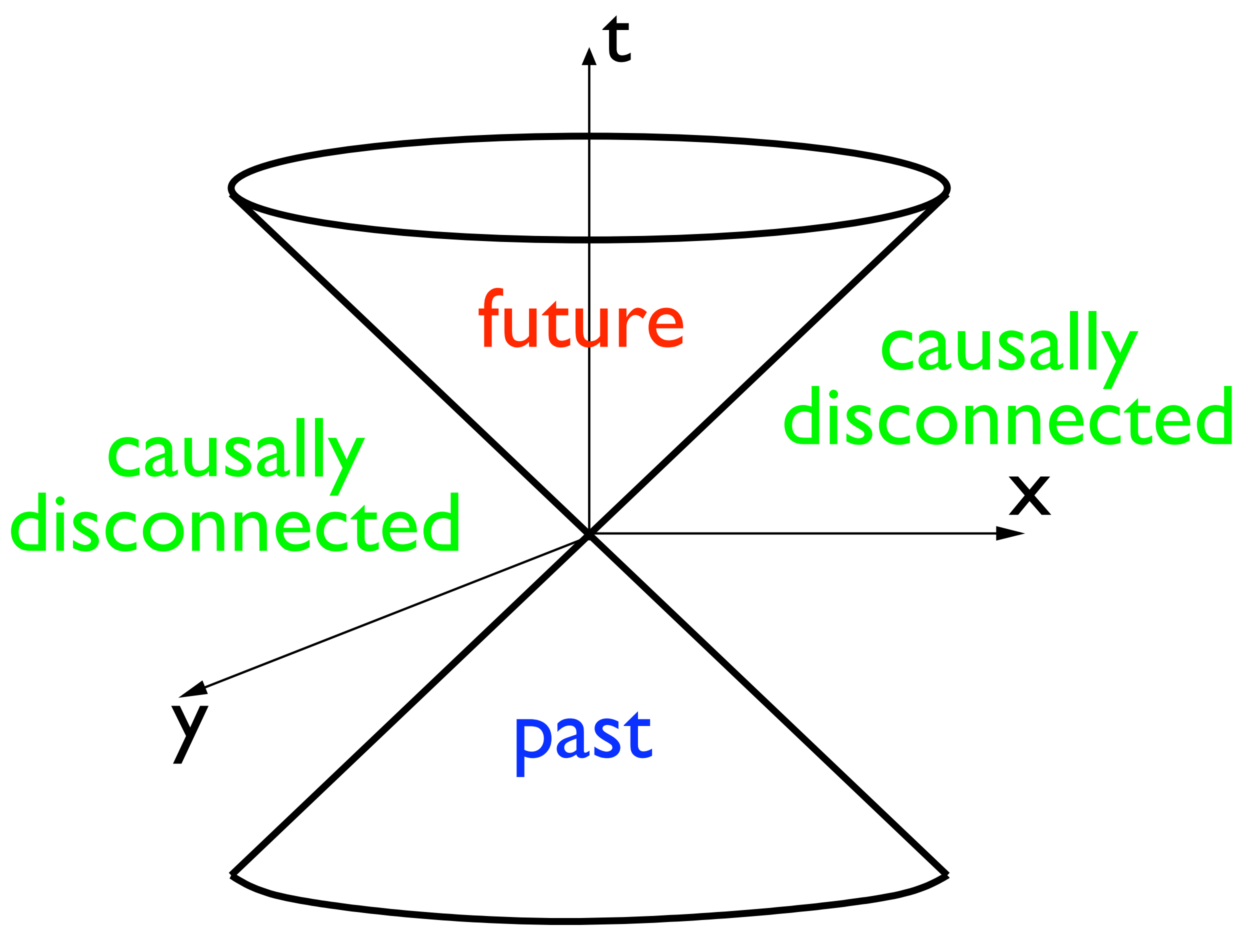


Superliminal Einstein-aether

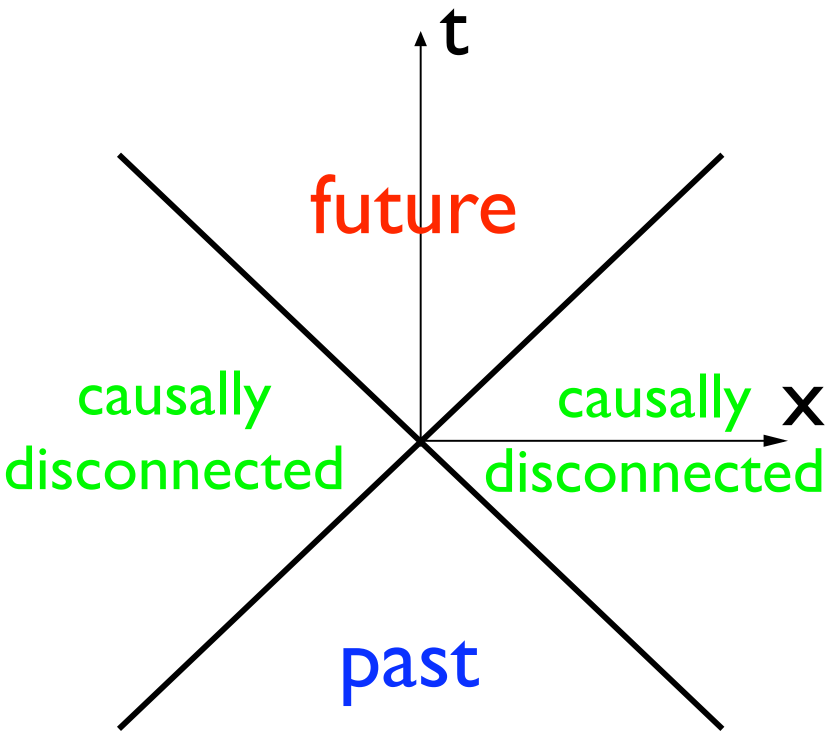


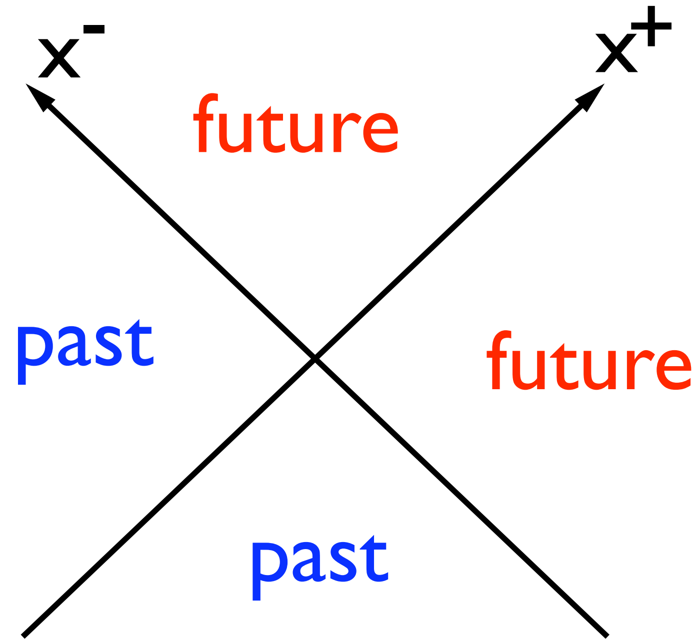
Instead of spontaneous LS breaking a *Lorentz invariant* model with *superluminal propagation*

Lorentz invariant causal structure: $d > 2$



Lorentz invariant causal structures: 2d





Positive-definite Hamiltonian for evolution along x^+

$$H_{x^+} = \int dx^- \frac{\beta}{2g^2}\Big[(\partial_+\psi)^2 \mathrm{e}^{2\psi} + (\partial_-\psi)^2 \mathrm{e}^{-2\psi}\Big]$$

Hints at integrability: $\beta_{(-)} = 0$

$$S = \int dx^+ dx^- \frac{1}{g^2}\left[\partial_+\psi\partial_-\psi + \frac{\beta}{2}(\partial_+\psi)^2 \mathrm{e}^{2\psi}\right]$$

- symmetry:

$$\psi(x^+, x^-) \mapsto \psi(f(x^+), g(x^-)) - \frac{1}{2}\log f'(x^+) + \frac{1}{2}\log g'(x^-)$$

$$f = \frac{ax^+ + b}{cx^+ + d}$$

arbitrary

N.B. half of conformal symmetry

- Infinite number of integrals of motion $\partial_+ q = 0$

$$q = (\partial_-\psi)^2 - \partial_-^2\psi - \frac{\beta}{2}\partial_-\partial_+\psi\,\mathrm{e}^{2\psi}$$

- General classical solution
- Renormalizable by normal ordering

Hints at integrability: general case

Equation of motion:

$$2\partial_+\partial_-\psi+\beta_{(+)}\big(\partial_+^2\psi+(\partial_+\psi)^2\big)\mathrm{e}^{2\psi}+\beta_{(-)}\big(\partial_-^2\psi-(\partial_-\psi)^2\big)\mathrm{e}^{-2\psi}=0$$

equivalent to zero-curvature condition:

$$[\partial_+ + V, \partial_- + U] = 0$$

$$U=\sigma_+ + \frac{1}{\lambda-\lambda_+}\mathrm{e}^{-a\sigma_-}\left(\frac{\lambda_+^2\sigma_+}{\lambda_+-\lambda_-}\right)\mathrm{e}^{a\sigma_-}+\frac{1}{\lambda-\lambda_-}\mathrm{e}^{-b\sigma_-}\left(\frac{-\lambda_-^2\sigma_+}{\lambda_+-\lambda_-}\right)\mathrm{e}^{b\sigma_-}$$

$$V=\frac{1}{\lambda-\lambda_+}\mathrm{e}^{-a\sigma_-}\left(\frac{\lambda_+ s\sigma_+}{\lambda_+-\lambda_-}\right)\mathrm{e}^{a\sigma_-}+\frac{1}{\lambda-\lambda_-}\mathrm{e}^{-b\sigma_-}\left(\frac{-\lambda_- s\sigma_+}{\lambda_+-\lambda_-}\right)\mathrm{e}^{b\sigma_-}$$

$$a=-\partial_-\psi+\lambda_-\partial_+\psi\mathrm{e}^{2\psi} \qquad b=-\partial_-\psi+\lambda_+\partial_+\psi\mathrm{e}^{2\psi} \qquad s=\mathrm{e}^{-2\psi}$$

$$\sigma_+=\begin{pmatrix}0&1\\0&0\end{pmatrix}$$

$$\beta_{(+)}=\frac{-2\lambda_+\lambda_-}{\lambda_++\lambda_-} \qquad \beta_{(-)}=\frac{-2}{\lambda_++\lambda_-}$$

$$\sigma_-=\begin{pmatrix}0&0\\1&0\end{pmatrix}$$

Coupling to gravity

Start from the Einstein-aether form of the action

$$S_{gr} = -\frac{1}{2\pi\kappa}\int d^2x\sqrt{-g}\left(R+\mu^2\right)+S_{EA}(g_{\mu\nu},V_\mu)$$

Fix conformal gauge ➡ Einstein-aether sector contributes to the Liouville action as a single scalar boson

+ explicit coupling to the Liouville field

$$S = \tilde{S}_{EA}(\phi,\psi) + S_L(\phi) + \ldots$$

$$\tilde{S}_{EA} = \int d^2x\left\{\frac{1}{g^2}\partial_+\psi\partial_-(\psi-\phi)+\frac{\beta_{(+)}}{2g^2}\mathrm{e}^{2\psi-\phi}(\partial_+\psi)^2+\frac{\beta_{(-)}}{2g^2}\mathrm{e}^{\phi-2\psi}\left(\partial_-(\psi-\phi)\right)^2\right\}$$

Prospects

- Toy models for issues related to causality

  Weak coupling to ordinary (massive) fields

$$S_\chi = \int d^2x \left(\partial_+\chi\partial_-\chi - \frac{m^2\chi^2}{2} + \frac{\gamma_{(+)}}{2}(\partial_+\chi)^2 \mathrm{e}^{2\psi} + \frac{\gamma_{(-)}}{2}(\partial_-\chi)^2 \mathrm{e}^{-2\psi} \right)$$

➡ apparent “a-causal effects”

- Toy models for extraction of information from black holes

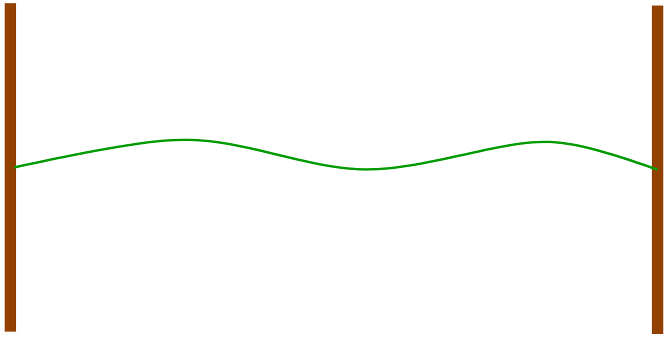
Prospects: more ambitious

Consider EA as (a part of) world-sheet action of a string

Possible action:

$$S = \int d^2x \left\{ \frac{1}{g^2} \left[\partial_+\psi \partial_-(\psi - \phi) - \partial_+ X^i \partial_- X^i \right] + \frac{\beta}{2g^2} \mathrm{e}^{2\psi - \phi} \left[(\partial_+\psi)^2 + (\partial_+ X^i)^2 \right] \right\} + S_L(\phi) + \dots$$

The resulting theory will be UV complete, contain gravity and possess unusual properties: a-causality ? non-locality ?



If it exists ...

At stake: holidays at α Centauri