ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Standard Model Higgs boson and cosmological inflation

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4th International Sakharov Conference

Based on:

- F. Bezrukov, M. S., Phys. Lett. B 659 (2008) 703
- F. Bezrukov, D. Gorbunov, and M. S., arXiv:0812.3622 [hep-ph]
- F. Bezrukov, A. Magnin, M. S., Phys. Lett. B 675 (2009) 88
- F. Bezrukov, M. S., arXiv:0904.1537 [hep-ph]

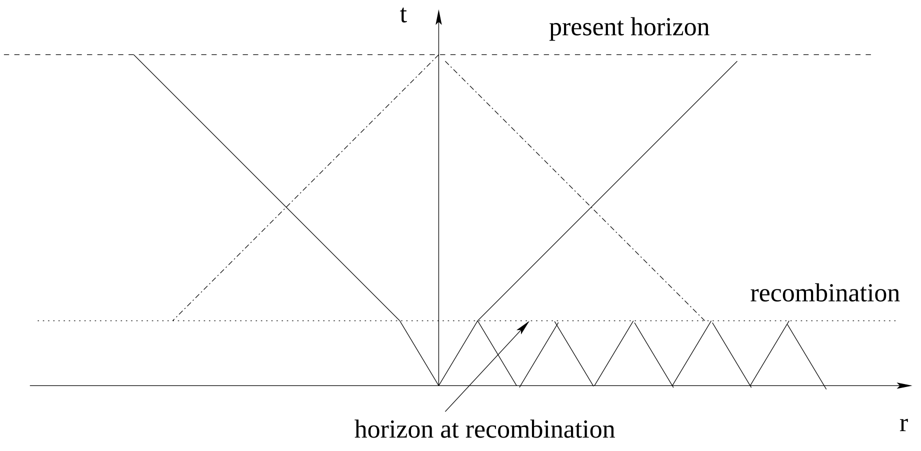
Outline

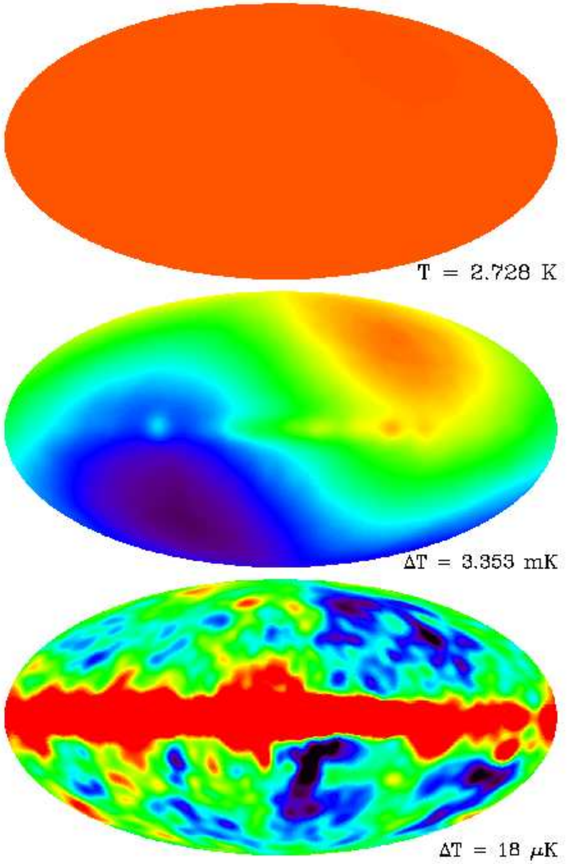
- Inflation and the inflaton
- Standard Model Higgs boson as inflaton
- CMB parameters in Higgs inflation
- SM as a valid effective theory up to the Planck scale
- Higgs mass from inflation
- Conclusions

Inflation and the inflaton

Important cosmological problems:

Horizon problem: Why the universe is so uniform and isotropic?



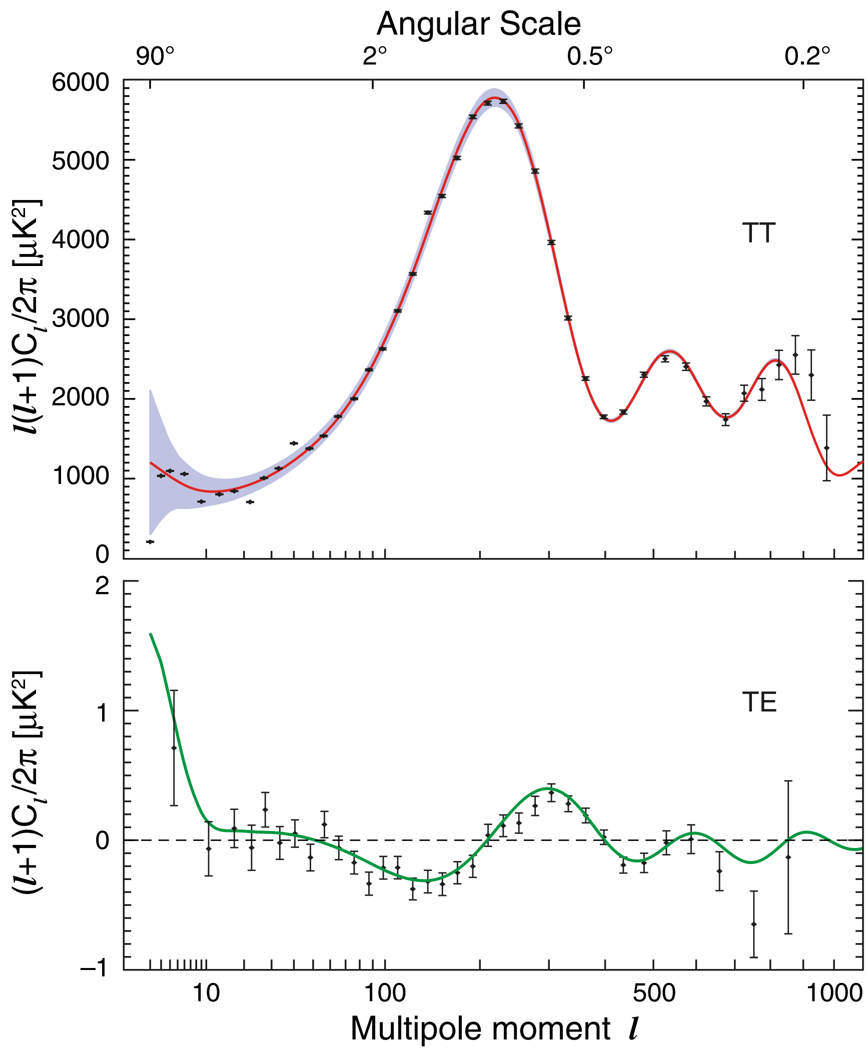


Expected fluctuations at $\theta \sim 1^o$:

$\delta T/T \sim 1$.

Observed fluctuations: $\delta T/T \sim 1O^{-5}$

Structure formation problem: What is the origin of cosmological perturbations and why their spectrum is almost scale-invariant?

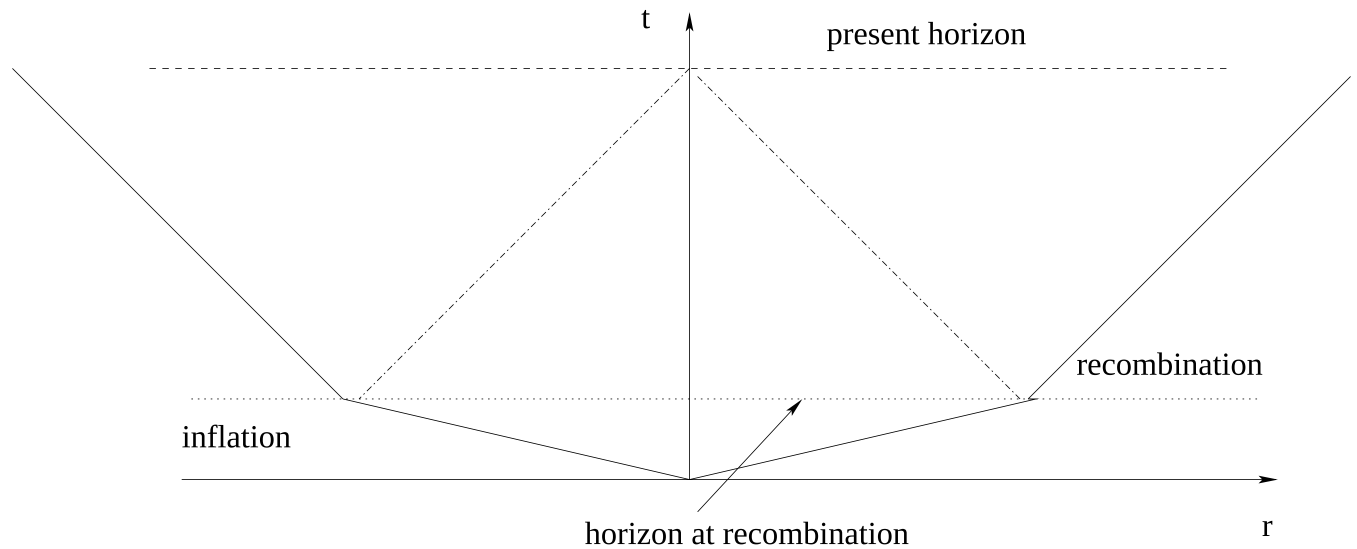


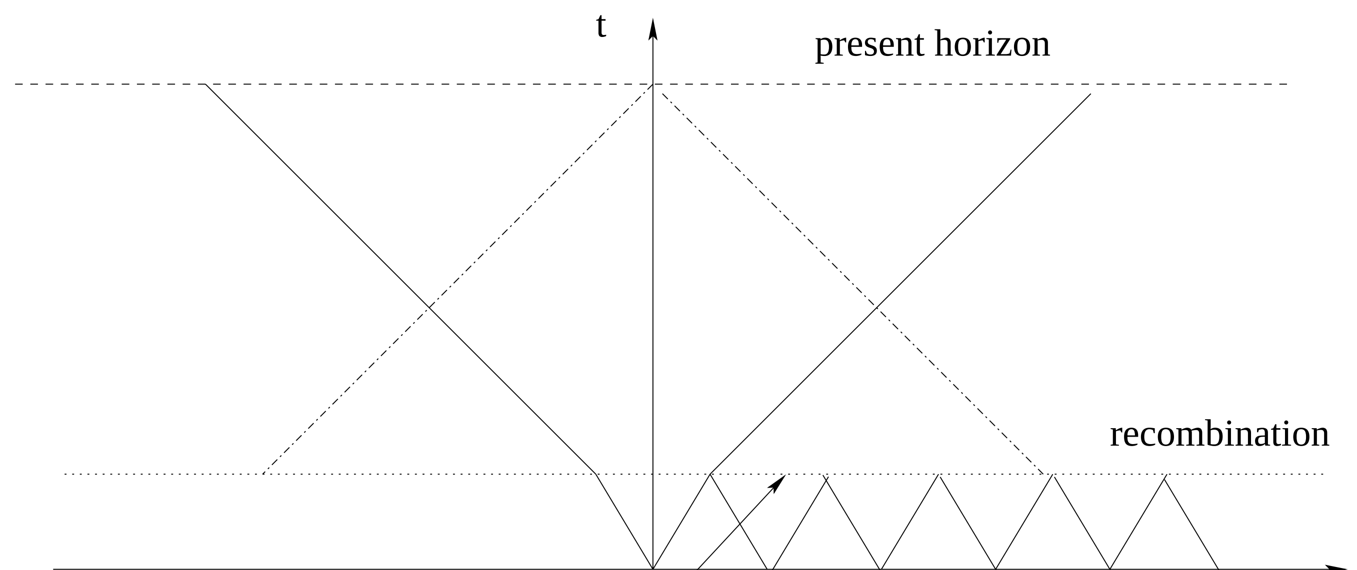
Sakharov peaks

Flatness problem: Why $\mathbf{\Omega}_M + \mathbf{\Omega}_\Lambda + \mathbf{\Omega}_{\mathbf{rad}}$ is so close to 1 now and was immensely close to 1 in the past?

All this requires enormous fine-tuning of initial conditions (at the Planck scale?) if the Universe was dominated by matter or radiation all the time!

Solution: Inflation = accelerated Universe expansion in the past





Mechanism: scalar field dynamics

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Why scalar?

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- Vector - breaking of Lorentz symmetry

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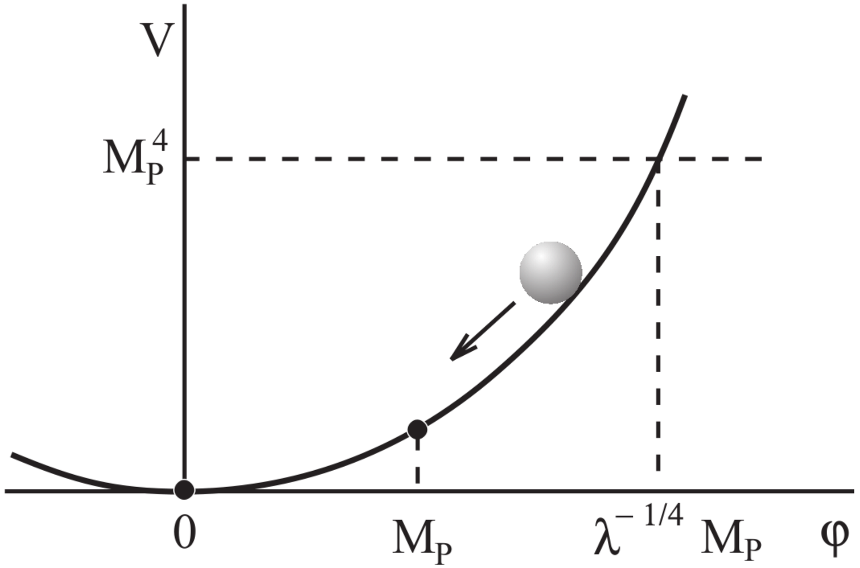
Why scalar?

- Vector - breaking of Lorentz symmetry
- Fermion - bilinear combinations are equivalent to scalar fields
- Uniform scalar condensate has an equation of state of cosmological constant and leads to exponential universe expansion.

“Standard” chaotic inflation

$$V(\phi) \quad = \quad \frac{1}{2}m^2\phi^2 \;+\; \frac{\lambda}{4}\phi^4$$



Almost flat potential for large scalar fields is needed! Linde

Required for inflation: (to get $\delta T/T \sim 10^{-5}$)

- quartic coupling constant $\lambda \sim 10^{-13}$:
- mass $m \sim 10^{13}$ GeV,

Present in the Standard Model: Higgs boson

- $\lambda \sim 1$, $m_H \sim 100$ GeV
- $\delta T/T \sim 1$

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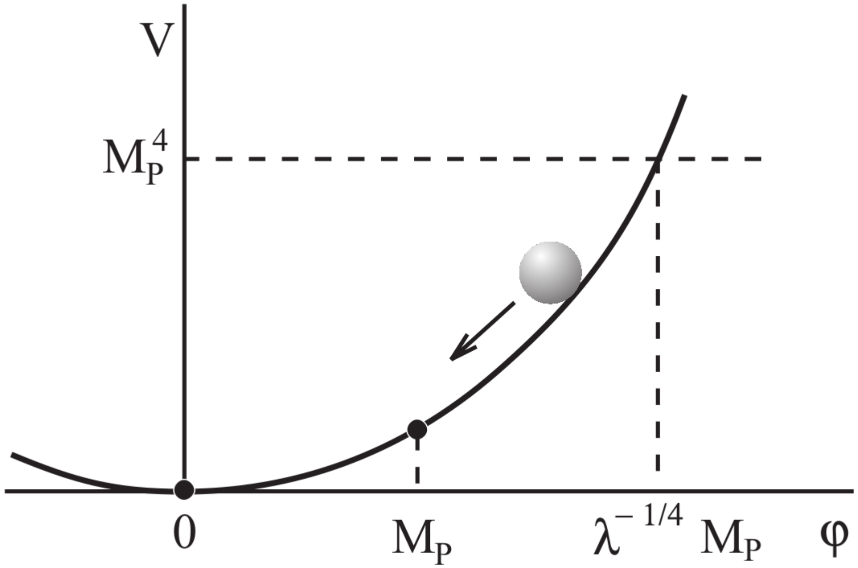

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New physics is required?

No - these conclusions are based on a theory with minimal coupling of scalar to gravity!:

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{M^2}{2} R + g_{\mu\nu} \frac{\partial^\mu h \partial^\nu h}{2} - \frac{\lambda}{4} \left(h^2 - v^2 \right)^2 \right\}$$

Extra term, necessary for renormalizability:

non-minimal coupling of scalar to gravity

$$\Delta S = \int d^4x \sqrt{-g} \left\{ -\frac{\xi h^2}{2} R \right\}$$

Feynman, Brans, Dicke,...

Standard Model Higgs boson as inflaton

Consider large Higgs fields h.

- Gravity strength: $M_P^{\text{eff}} = \sqrt{M_P^2 + \xi h^2} \propto h$
- All particle masses are $\propto h$

For $h > \frac{M_P}{\xi}$ (classical) physics is the same (M_W / M_P^{eff} does not depend on h)!

Existence of effective flat direction, necessary for successful inflation.

Formalism: go from Jordan frame to Einstein frame with the use of conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \; , \quad \Omega^2 = 1 + \frac{\xi h^2}{M_P^2}$$

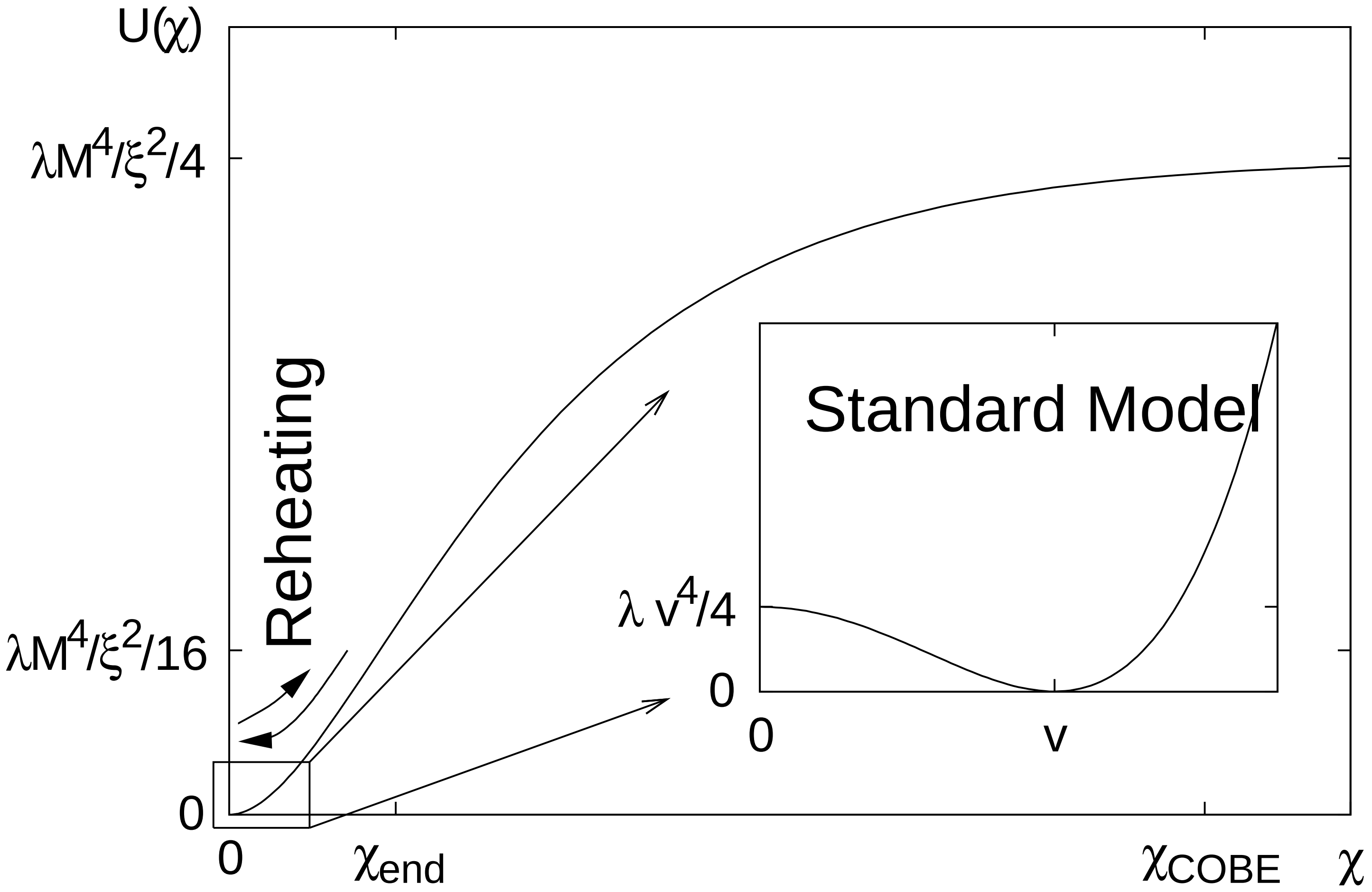
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{1}{\Omega(\chi)^4}\frac{\lambda}{4}h(\chi)^4 \right\}$$

Potential:

$$U(\chi) = \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } h < M_P/\xi \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } h > M_P/\xi \end{cases} .$$

Potential in Einstein frame



Inflaton potential and observations

If inflaton potential is known one can make predictions and compare them with observations.

- $\delta T/T$ at the WMAP normalization scale ~ 500 Mpc
- The value of spectral index n_s of scalar density perturbations

$$\left\langle \frac{\delta T(x)}{T} \frac{\delta T(y)}{T} \right\rangle \propto \int \frac{d^3k}{k^3} e^{ik(x-y)} k^{n_s-1}$$

- The amplitude of tensor perturbations $r = \frac{\delta\rho_s}{\delta\rho_t}$

These numbers can be extracted from WMAP observations of cosmic microwave background. Higgs inflation: one new parameter, $\xi \Longrightarrow$ two predictions.

Slow roll stage

COBE normalization $U/\epsilon = (0.027 M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{\rm COBE}}{0.027^2}} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v}$$

Connection of ξ and the Higgs mass!

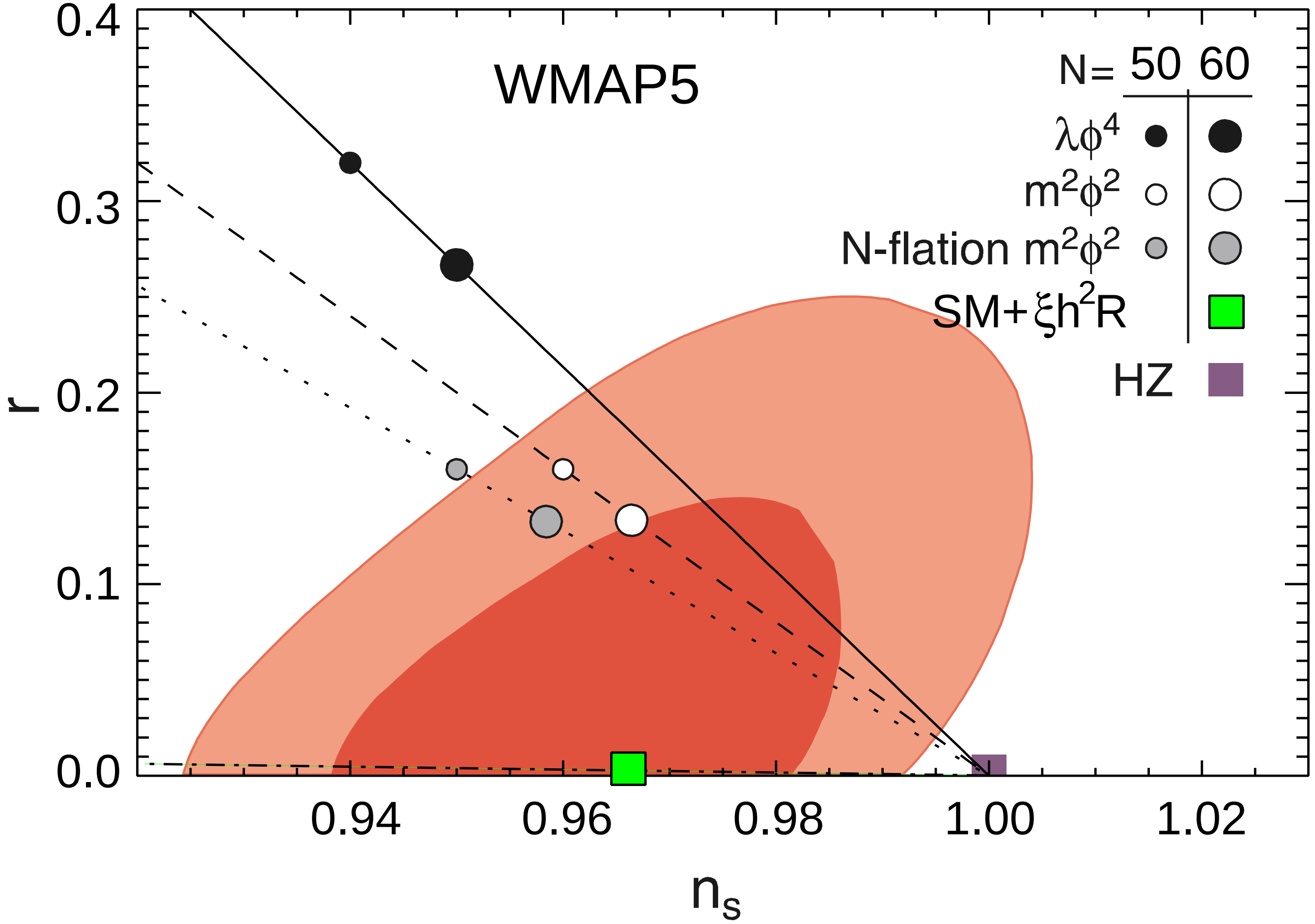
Number of e-folds of inflation at the moment h_N is $N \simeq \frac{6}{8}\frac{h_N^2 - h_{\rm end}^2}{M_P^2/\xi}$

Slow roll ends at $\chi_{\rm end} \simeq M_P$; and "begins" at $\chi_{60} \simeq 5M_P$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{dU/d\chi}{U}\right)^2, \quad \eta = M_P^2\frac{d^2U/d\chi^2}{U}$$

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon$$

CMB parameters—spectrum and tensor modes



Earlier works: non-minimal coupling of scalars in GUTs, etc:

- B. Spokoiny ’84
- D. Salopek, J. Bond and J. Bardeen ’89
- R. Fakir and W. G. Unruh ’90
- A. O. Barvinsky and A. Y. Kamenshchik ’94, ’98
- E. Komatsu and T. Futamase ’99
- S. Tsujikawa and B. Gumjudpai ’04

Computation of spectral indexes gives the same results in Einstein and Jordan frames.

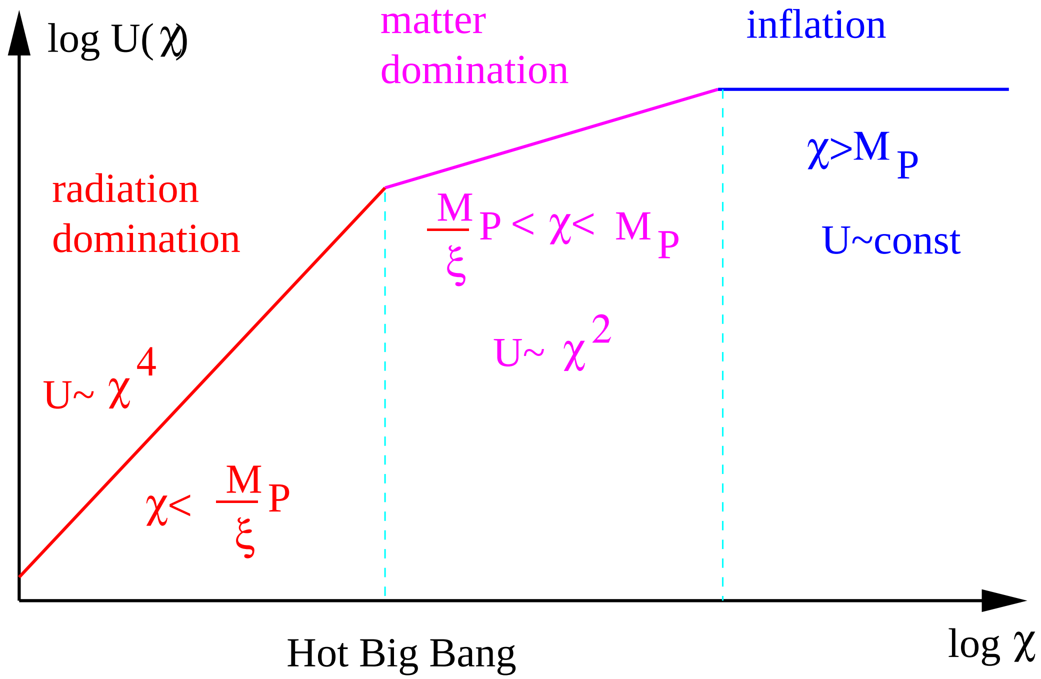
Life after inflation

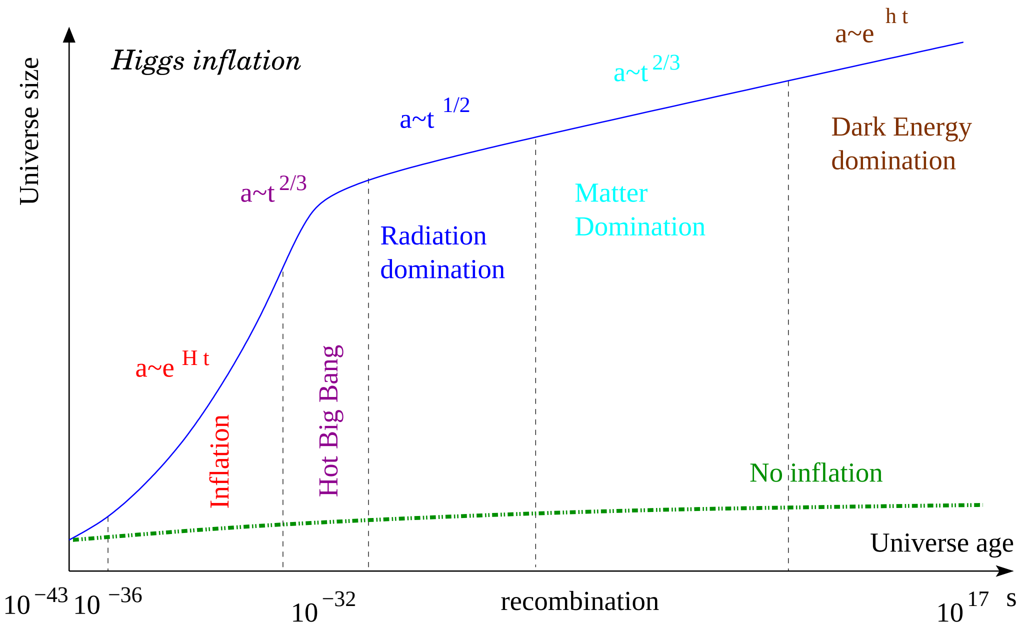
Two different stages:

- For scalar field $M_P > \chi > \frac{M_P}{\xi}$ the potential for χ is essentially *quadratic*, $m_\chi^2 \sim \lambda M_P^2/\xi^2$. Exponential expansion of the Universe is changed to the power low, corresponding to matter domination. Particle creation takes place when χ passes through zero.
- After $\mathcal{O}(\xi)$ oscillations the scalar field reaches $\chi \simeq \frac{M_P}{\xi}$. The energy is transferred to other fields of the SM, and the radiation-dominated epoch starts,

$$T_r \simeq (3.3 - 8.3) \times 10^{13}\ \mathrm{GeV}.$$

See also J. Garcia-Bellido, D. G. Figueroa, J. Rubio '08


log U(χ)
matter
domination
inflation
radiation
domination
$\frac{M}{\xi}P < \chi < M_P$
$\chi > M_P$
U~const
U~ χ^4
U~ χ^2
$\chi < \frac{M}{\xi}P$
Hot Big Bang
log χ



Higgs inflation
Universe size
a~e h t
a~t 2/3
a~t 1/2
a~t 2/3
Dark Energy domination
Matter Domination
Radiation domination
a~e H t
Inflation
Hot Big Bang
No inflation
Universe age
10 −43 10 −36
10 −32
recombination
10 17 s

Higgs mass from inflation: qualitative argument

Previous consideration tells nothing about the Higgs mass: change λ as $\propto \xi^2$ - no modifications!

However: λ is not a constant, it depends on the energy. Typical scale at inflation $\sim M_P/\sqrt{\xi}$.

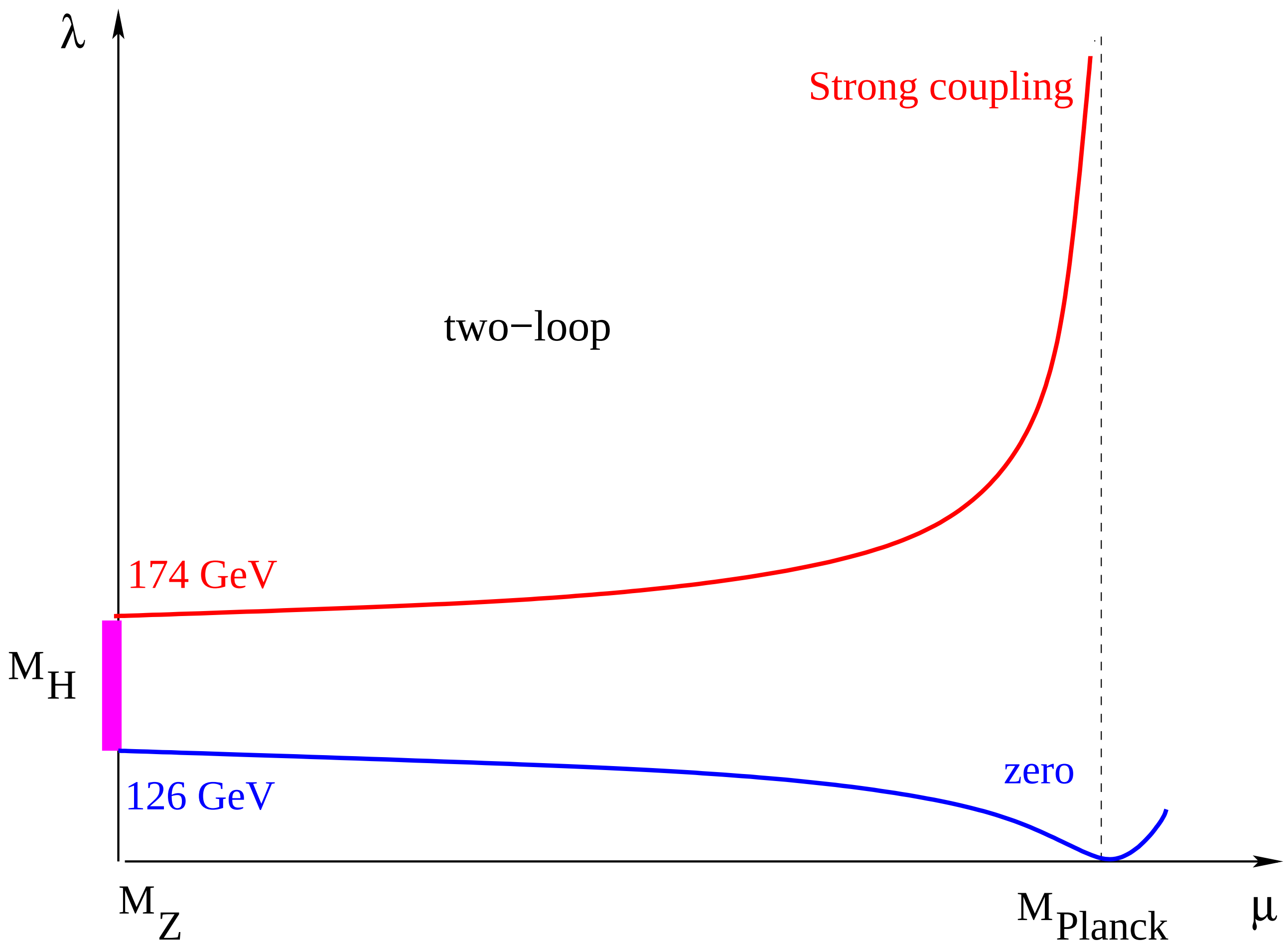
Therefore, SM must be a valid quantum field theory up to the Inflation (or, to be on safe side, up to the Planck scale).

$$m_{\min} < m_H < m_{\max}$$

$$m_{\min} = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 0.6] \text{ GeV}$$

$$m_{\max} = [173.5 + \frac{m_t - 171.2}{2.1} \times 1.1 - \frac{\alpha_s - 0.1176}{0.002} \times 0.3] \text{ GeV}$$

Behaviour of the scalar self-coupling



For $m_H > m_{\max}$: Landau pole for energies $E = E_{\rm Landau} < M_P$ – quantum field theory is inconsistent for $E > E_{\rm Landau}$.

For $m_H < m_{\min}$: Electroweak vacuum is unstable: there is a lower ground state at $\phi < M_P$.

Higgs mass from inflation: computation

Electroweak theory in the inflationary region, for

$h \sim M_P/\sqrt{\xi}, \;\; h \gg M_P/\xi$:

Take the SM, freeze the radial mode of the Higgs field, and add to Lagrangian almost massless and almost non-interacting scalar: chiral SM.

Why the Higgs decouples?

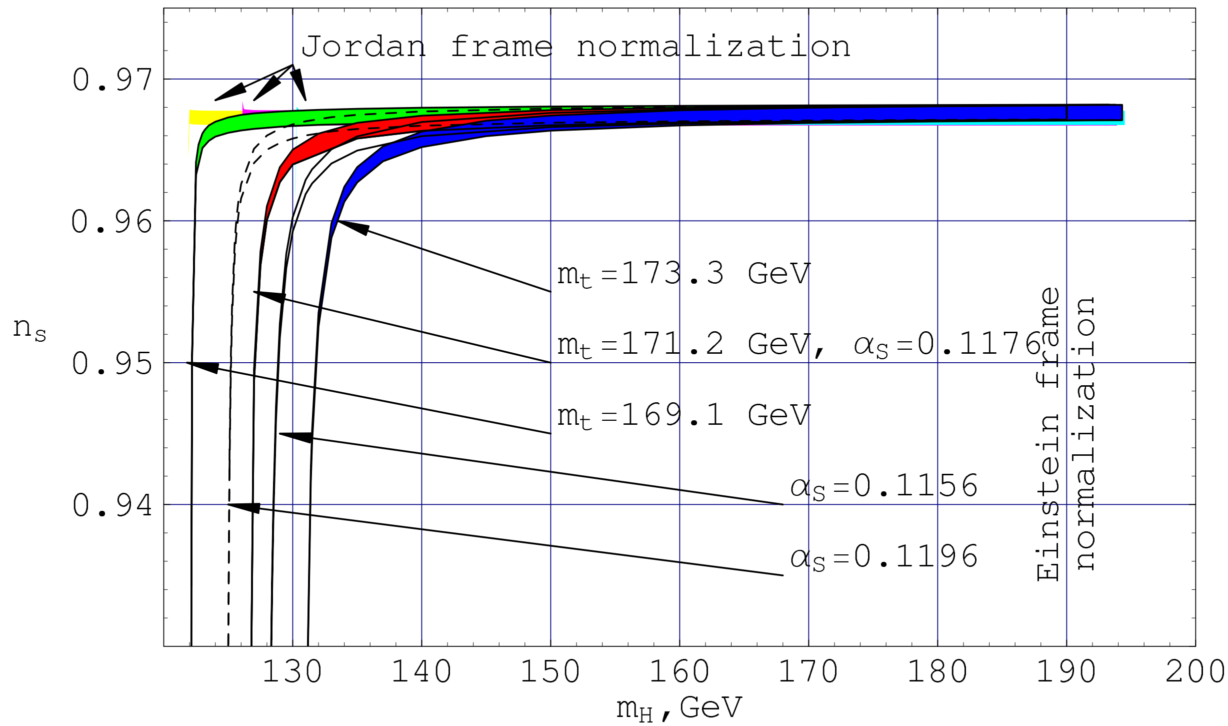
Einstein frame: masses of all the particles

$$M_W, ..., m_t \propto v = \frac{h}{\Omega(h)} \rightarrow \textbf{const for } h \rightarrow \infty$$

The procedure for computations of inflationary parameters

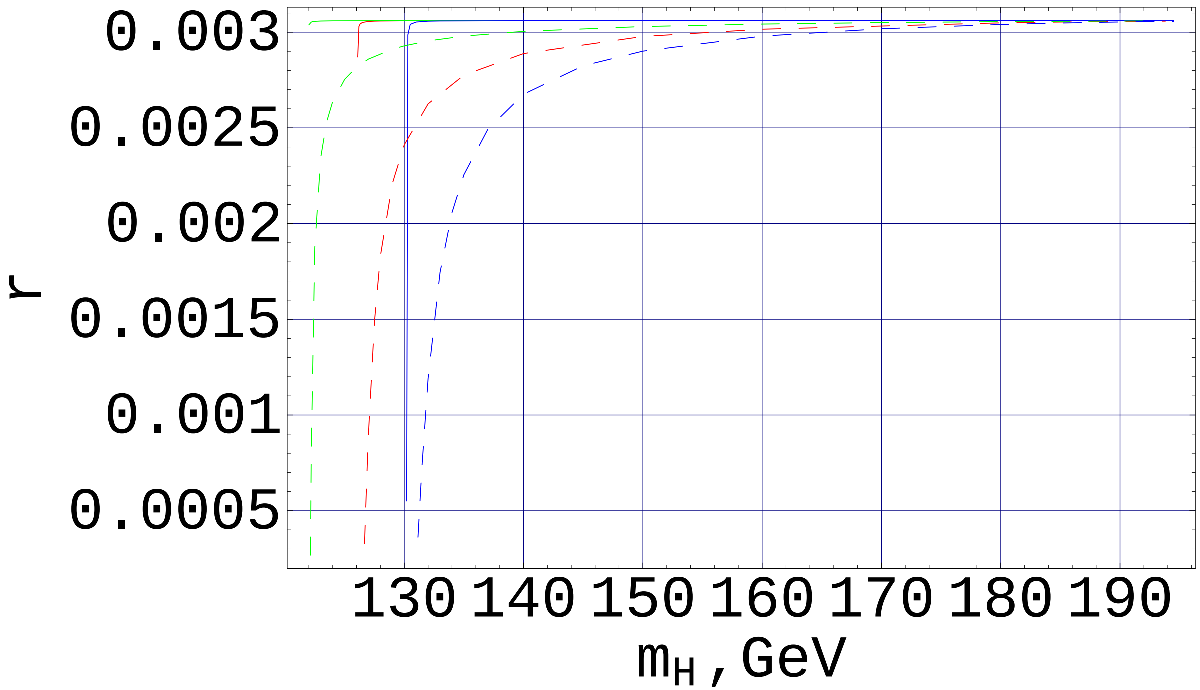
- Compute the effective potential in the inflationary region (tree, one-loop, two-loop,... with the use of the chiral SM
- Choose the normalization point μ to minimize higher order terms (normally, $\mu \sim M_W$ or M_Z or m_t)
- To find values of different coupling constants in inflationary region, solve one-loop, two-loop ... RG equations, getting initial conditions from tree, one-loop, two-loop,.. relations mapping physical SM parameters to couplings.
- Find ξ from COBE normalization and compute n_s and r as a function of the Higgs mass

Two-loop results



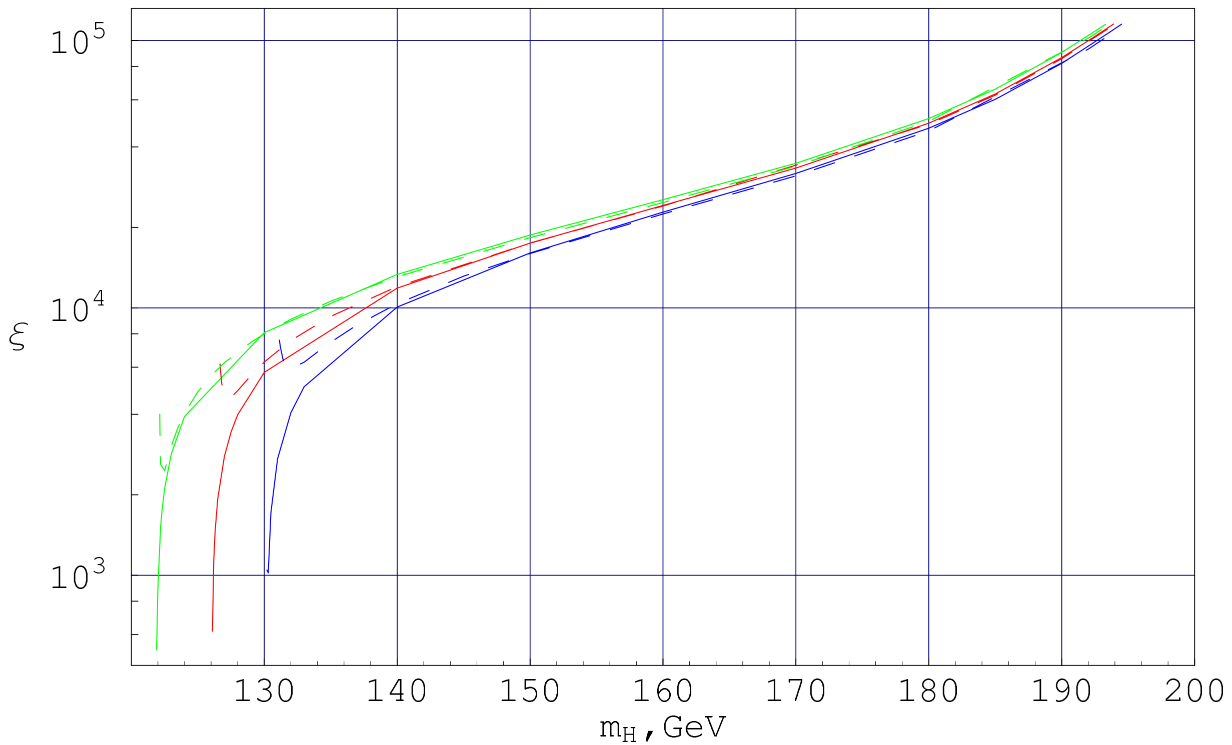
Nearly horizontal coloured stripes correspond to the normalization prescription I. Green, red, and blue stripes give the result with normalization prescription II for different m_t and $\alpha_s = 0.1176$, two white regions correspond to different α_s and $m_t = 171.2\text{GeV}$. The width of the stripes corresponds to changing the number of e-foldings between 58 and 60, or approximately one order of magnitude in reheating temperature.

Two-loop results



Tensor-to-scalar ratio r depending on the Higgs mass m_H, calculated with the RG enhanced effective potential. Nearly horizontal solid lines correspond to the normalization prescription I. Green, red, and blue dashed lines give the result with normalization prescription II for $m_t = 169.1, 171.2, 173.3$ GeV. Dependence on the number of e-foldings is very small.

Two-loop results



$\boldsymbol{\xi}$ at the scale M_P/ξ depending on the Higgs mass m_H for $m_t = 171.2, 169.1, 173.3$ GeV (from upper to lower graph). Solid lines correspond to prescription I, dashed—to prescription II. Changing the e-foldings number and error in the WMAP normalization measurement introduce changes invisible on the graph.

Cosmological constraint on the Higgs mass

1 loop computation

$$m_{\min} = [136.7 + (m_t - 171.2) \times 1.95]\ \mathrm{GeV}$$
$$m_{\max} = [184.5 + (m_t - 171.2) \times 0.5\ \ ]\ \mathrm{GeV}$$

2 loop computation

$$m_{\min} = [126.1 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \text{GeV}\ ,$$
$$m_{\max} = [193.9 + \frac{m_t - 171.2}{2.1} \times 0.6 - \frac{\alpha_s - 0.1176}{0.002} \times 0.1]\ \text{GeV}\ .$$

Main effect - the window for the Higgs mass is wider. Theoretical uncertainty ± 1.3 GeV. Spectral index behaviour is the same in one and two loops.

Experimental constraints on the Higgs mass

Direct searches

LEP limit: $m_H > 114.4$ GeV, 95% C.L.

Tevatron experiments CDF and D0: the region

$160\ \mathrm{GeV} < m_H < 170\ \mathrm{GeV}$ is excluded, 95% C.L.

The LEP Electroweak Working Group:

The preferred value:

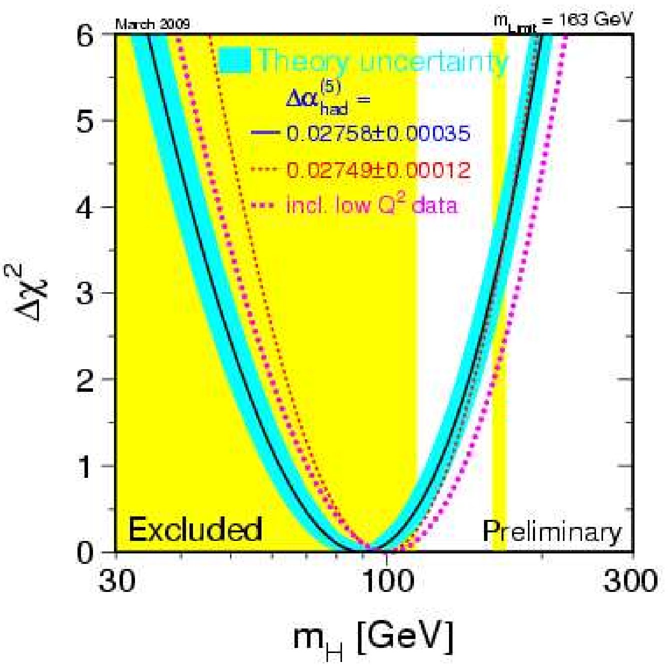
$m_H = 90^{+36}_{-27}$ GeV, 68% C.L.

Upper limit:

$m_H < 163$ GeV, one-sided 95% C.L.

Upper limit accounting for LEP result:

$m_H < 191$ GeV, one-sided 95% C.L.



Other works

- Result of A. De Simone, M. Hertzberg and F. Wilczek '08:
  Higgs inflation works if

$$m > [125.7 + \frac{m_t - 171}{2} \times 3.8 - \frac{\alpha_s - 0.1176}{0.0020} \times 1.4]\ \mathrm{GeV}$$

- Result of A. Barvinsky, A. Kamenshchik, C. Kiefer, A. Starobinsky, and C. Steinwachs '09:
  $m_{\min} = 124$ GeV, $m_{\max} = 180$ GeV
  (earlier computation with different number: A. Barvinsky, A. Kamenshchik, A. Starobinsky '08)

All 2-loop computation of $m_{\min}$ are in good agreement with each other. Discrepancy - in behaviour of n_s and r.

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- This region is somewhat wider that the region of validity of the SM all the way up to the Planck scale $M_H \in [126.3, 174]$ GeV (for central values of m_t and α_s).

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- Crucial experimental test - the LHC
- Crucial cosmological test - precise measurements of cosmological parameters $n_s, \quad r$

Extras

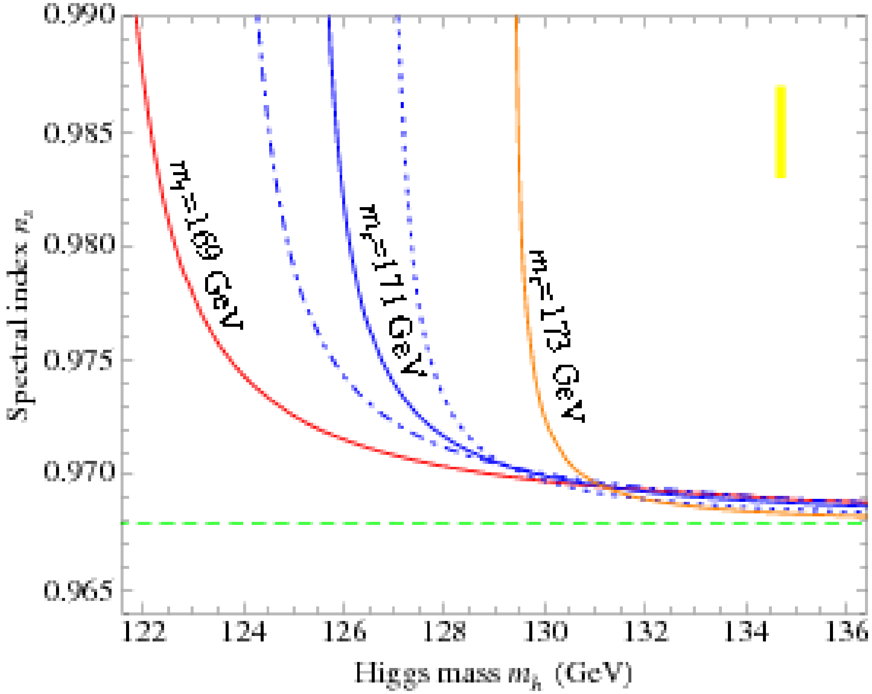
Subtle point: renormalization

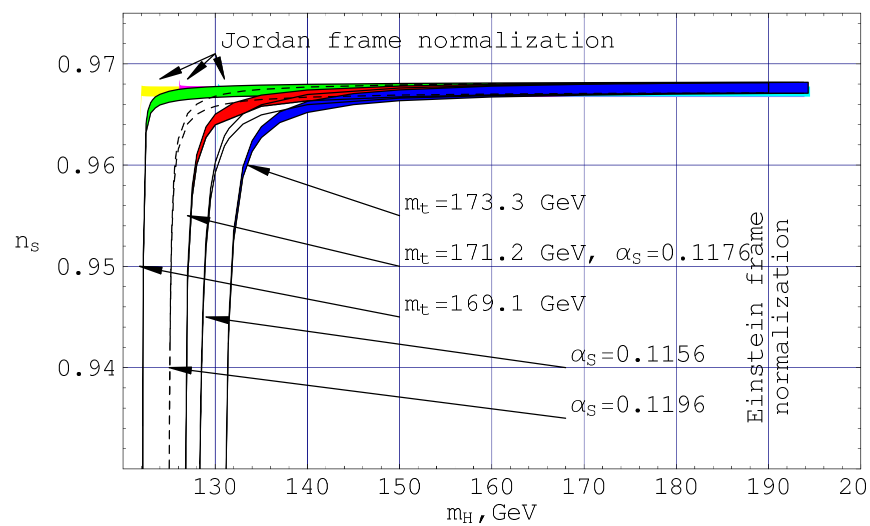
The straightforward computation in Einstein or Jordan frames leads to different results. μ in the different frames

	I BS	II BKS
Jordan frame	$M_P^2 + \xi h^2$	M_P^2
Einstein frame	M_P^2	$\frac{M_P^4}{M_P^2 + \xi h^2}$

The prescription I is “standard” (field-independent) in the Einstein frame, whereas the prescription II is “standard”(field-independent) in the Jordan frame. To be fixed by (unknown) physics at the Planck scale.

Comparison with other works





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Bezrukov, MS

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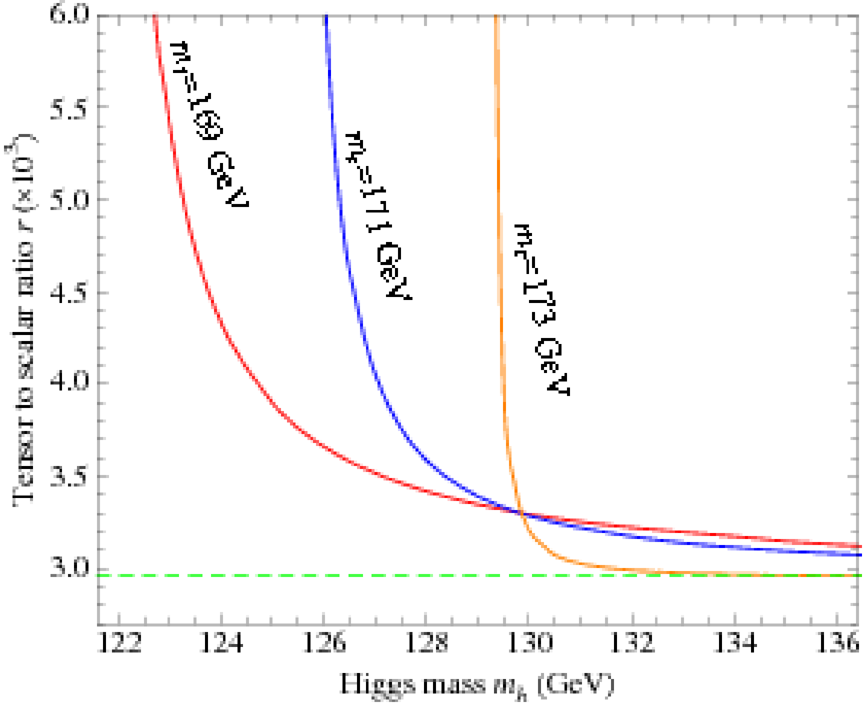

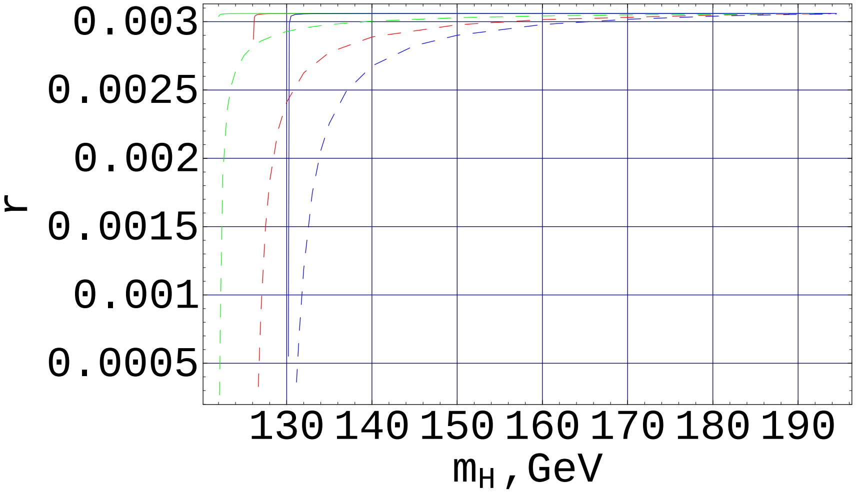

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$F(R)$ inflation

Starobinsky '80: keep the particle physics intact but modify the pure gravitational action:

$$L_G = \frac{M_P^2}{2} R + \alpha R^2$$

Equivalent to standard gravity + scalar field σ with potential

$$U(\sigma) = \frac{M_P^4}{16\alpha} \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\sigma}{M_P}}\right)^2$$

From COBE normalisation: $\alpha = 6 \times 10^8$. Spectral indices are the same as for the Higgs inflation. Reheating temperature $T_r \simeq 3 \times 10^9$ GeV. Inflaton mass $m_\sigma \simeq 3 \times 10^{13}$ GeV.

Fine-tuning problem - Higgs boson mass is driven up to 10^7 GeV by radiative corrections.