Deviation of AdS Minimal Area from BDS Extrapolation at a Wavy Circle

with D. Galakhov, A. Mironov and A. Morozov (ITEP)

	GIMM	arXiv : 0812.4702
	IMM2	arXiv : 0803.1547
see also	IMM	arXiv : 0712.0159
	IM8	arXiv : 0712.2316

Talk at the 4th Sakharov conference, Moscow, May 22, 2009

I) Introduction

- Why N=4 SYM ?

⇒
- simplest QFT
- the most basic realization of gauge/gravity correspondence

⇒
- may turn out to be exactly solvable
- applications which have been fruitful
- unified theory of 3+1 forces not anticipated 20 years ago

- Conjecture :

  N=4 SYM and IIB Superstrings in $AdS_5 \times S^5$ are equivalent

  the conjecture assumed in my talk

- A new approach to AdS/CFT by Alday-Maldacena '07

- Scattering amp. as opposed to correlators and their anomalous dim.

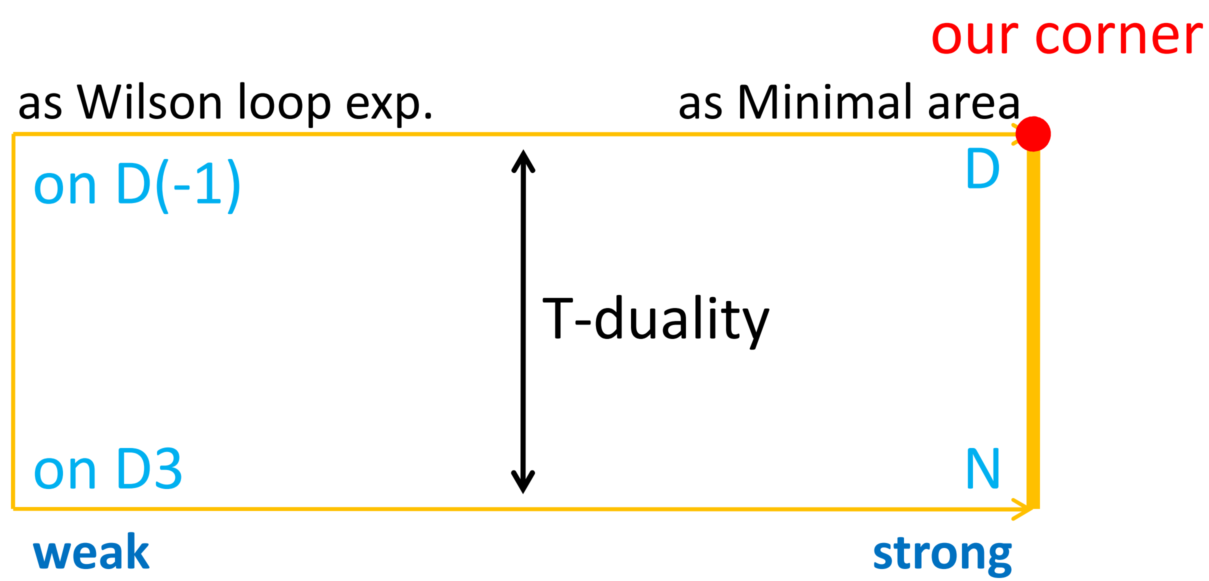
- Semiclassical string cal. as opposed to SUGRA appx.

- Situation

planar

representation

gauge coupling



- Things known before our work

First introduce, with regard to n-gluon amplitude,

$\sqcap$; polygon formed by a set of gluon momenta

$M^{(1)}$; one-loop result of the scalar fn from MHV amp.

also

$D_{\sqcap}$; log of the abelian Wilson loop average with $\sqcap$;

$$D_{\sqcap} = \oint_{\sqcap} \oint_{\sqcap} \frac{dy^{\mu} dy'_{\mu}}{(y - y')^{2+\epsilon}}$$

- $M^{(1)} = D_{\sqcap}$ by explicit computation

Drummond, Korchemsky, Sokatchev; Brandhuber, Heslop, Travaglini; Mironov, Morozov, Tomaras.

c.f. Gorsky's talk, Makeenko's talk

- The BDS (Bern-Dixon-Smirnov) exponentiation $e^{\tilde{\kappa} D_{\sqcap}}$ represents a substancial part of the complete amplitude $\mathcal{A}_n$, but by now nonvanishing remainder fn confirmed numerically, starting at n=6, L=2 loop at weak coupling. BDKKRSVV 08031465
DHKS 08031466

- Correspondence between the scalar MHV function and the Wilson loop appears to be valid beyond one-loop.

- our comparison at the wavy circle
in the strong coupling limit - $A_{\sqcap}(\text{area})$ v.s. $\kappa D_{\sqcap}$

$e^{\kappa\sqrt{\lambda}D_{\sqcap}}$; extrapolation of BDS to strong coupling

$e^{\sqrt{\lambda}A_{\sqcap}}$; semiclassical string amp. $A_{\sqcap}$ minimal area

reveals disparity analytically

Contents

I) Introduction

II) Nambu-Goto equation and the linearized form

III) Method of computation for wavy boundaries

IV) Our results : $A_{\sqcap} \not\sim \kappa D_{\sqcap}$

explicit form of $D^{(m|n)}$, $A^{(2,2)}$

$$\triangle_{\text{IMM}} \psi = 0$$

V) More recent developments

$$\triangle_{\text{IMM}} \equiv \triangle_0 - \mathcal{D}^2 + \mathcal{D}$$

II)

- AdS / CFT duality ; $\sqrt{\lambda} \equiv \sqrt{g^2 N} = \frac{R^2}{\alpha'}, \ \frac{1}{N} \sim g_s$

- work on the Euclidean worldsheet $\xi^1 = y_1 \ , \quad \xi^2 = y_2$

- The 1st ansatz ; $y_3 = 0 \quad \cdots$ ①

$$S_{E,NG} = \frac{\sqrt{\lambda}}{2\pi} \int dy_1 dy_2 \sqrt{\det H} \ , \quad H_{ij} = \frac{1}{r^2}(\delta_{ij} - \partial_i y_0 \partial_j y_0 + \partial_i r \partial_j r)$$

recognize this as $f(\lambda) \overset{\lambda \text{ large}}{\sim} \sqrt{\lambda}$

- The 2nd ansatz ; $1 = y_\mu y^\mu + r^2 \ \Leftrightarrow \ Y^4 = 0 \quad \cdots$ ②

- ① and ② contain the Alday-Maldacena rhombus solution.
  (y^0 shift involved)

- Eq. of motion

$$\delta y_0 \;:\; \partial_1\left(\frac{H_{22}}{r^2\sqrt{\det H}}\partial_1 y_0\right)+\partial_2\left(\frac{H_{11}}{r^2\sqrt{\det H}}\partial_2 y_0\right) - \partial_1\left(\frac{H_{21}}{r^2\sqrt{\det H}}\partial_2 y_0\right)-\partial_2\left(\frac{H_{12}}{r^2\sqrt{\det H}}\partial_1 y_0\right)=0$$

δr : similar one

In practice, eliminate r^2 through $S_{E,NG}$ and ②

- linearization w.r.t. y_0

$$\triangle_{\rm IMM} y_0 = 0 \;, \text{where } \triangle_{\rm IMM} = \triangle_0 - \mathcal{D}^2 + \mathcal{D}$$
$$\triangle_0 = 4\partial\bar\partial,\; \mathcal{D} = z\partial + \bar z\bar\partial$$

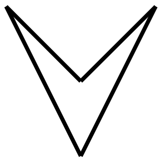
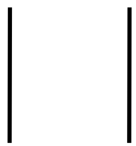
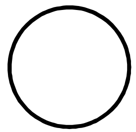
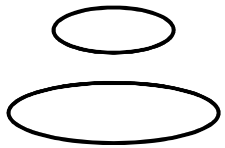
general regular solution : $y_0 = \sum_{k\geq 0} Re(\alpha_k z^k) g_k(x = z\bar z)$

A complete set of harmonics :

$$g_k(x) = \frac{1+k\sqrt{1-x}}{(1+\sqrt{1-x})^k}\,,\quad \tilde g_k(x) = \frac{1-k\sqrt{1-x}}{(1-\sqrt{1-x})^k}$$

- linearization → $\triangle_{\rm IMM}$ appears to be generic

III) ▪ Exact solutions (minimal surfaces named by their bndaries)

rhombus (light like)		$\kappa_{\square} = 1$	Alday-Mal 1
infinite strip (planar)		$\kappa_{\text{strip}} = 4\frac{(2\pi)^2}{(\Gamma(1/4))^4}$	Alday-Mal 3
circle (planar)		$\kappa_{\text{circle}} = \frac{3}{\pi}$	Berenstein, Corrado, Fischler, Maldacena ①+②+ “ $y^0 = 0$ ”
two circles (parallel)			Olesen-Zarembo
		non universality not well understood	

▪ how to deal with the issue $A_{\sqcap} \overset{?}{\approx} \kappa D_{\sqcap}$ fruitfully

our proposal : make them WAVY

by introducing ∞ly many parameters

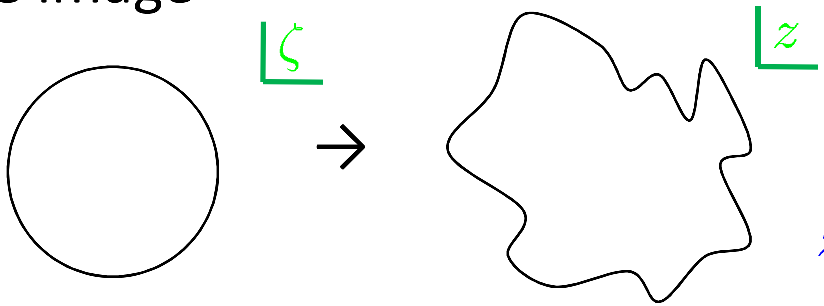
- formulation

  - the circle solution $r^2 = 1 - y_i^2$
  - deform the circle into

    by the conformal map $z = H(\zeta)$

$$\partial z = 1 + \sum_{k=1}^{\infty} k h_k \zeta^{k-1} \equiv \partial H \equiv 1 + \partial h \qquad \bar{\partial} z = 0$$

  - unit circle = pre image



bdd by Π

$z = y_1 + i y_2$

  - find $a(\zeta, \bar{\zeta}; h, \bar{h})$ s.t. $r^2(z, \bar{z}) = 1 - \zeta\bar{\zeta} + a(\zeta, \bar{\zeta})$

$$a|_{|\zeta|=1} = 0$$

- action

$$S_{NG}[a,h] = \frac{\sqrt{\lambda}}{2\pi}\int d^2\zeta \frac{1}{r^2+\mu^2}\sqrt{|\partial H|^2(|\partial H|^2+4\partial r\bar{\partial}r)}$$

μ ; regularization

- once again, eq. of motion ;

$$0 = \partial\left(\frac{\partial\mathcal{L}}{\partial(\partial a)}\right)+\bar{\partial}\left(\frac{\partial\mathcal{L}}{\partial(\bar{\partial}a)}\right)-\frac{\partial\mathcal{L}}{\partial a} = \frac{1}{4(1-\zeta\bar{\zeta})^{3/2}}\left\{\triangle_{\text{IMM}}(a(\zeta,\bar{\zeta}))+R(a;h,\bar{h})\right\}$$

and

$$a = a^{(1)} + a^{(2)} + \cdots$$

- Iterative construction of the solution ;

$$\triangle_{\text{IMM}}(a^{(1)}(\zeta,\bar{\zeta}) + \bar{\zeta}h(\zeta) + \zeta\bar{h}(\bar{\zeta})) = 0$$

$$\triangle_{\text{IMM}}(a^{(k)}(\zeta,\bar{\zeta})) = -R^{(k)}(\underset{\substack{\uparrow \\ a = a^{(1)}+\cdots+a^{(k-1)}}}{a};h,\bar{h}) \ , \quad k \geq 2$$

- invert by the method of constant variation or Green fn

IV) i) Notation and our results

$$A_{\sqcap}^{reg} = A_{\sqcap} - \frac{\pi L_{\sqcap}}{2\mu} + 2\pi = -3\pi \left(\sum_{m,n} (-)^{m+n} A^{(m|n)}_{k_1 \ldots k_m | \ell_1 \ldots \ell_n} h_{k_1+1} \ldots h_{k_m+1} \bar{h}_{\ell_1+1} \bar{h}_{\ell_n+1} \right)$$

$$D_{\sqcap}^{reg} = D_{\sqcap} - \frac{\pi L_{\sqcap}}{4\lambda} + \frac{\pi^2}{2} = -\pi^2 \left(\sum_{m,n} (-)^{m+n} D^{(m|n)}_{k_1 \ldots k_m | \ell_1 \ldots \ell_n} h_{k_1+1} \ldots h_{k_m+1} \bar{h}_{\ell_1+1} \bar{h}_{\ell_n+1} \right)$$

$k_j, \ \ell_j$ sums understood

$\mu, \ \lambda$; regularization parameters, $L_{\sqcap}$; the length of the contour ⊓

IMM2

$$A^{(1|1)}_{k|k} = \frac{(k+1)k(k-1)}{6} = D^{(1|1)}_{k|k}$$

$$A^{(2|1)}_{k_1,k_2|k_1+k_2} = \frac{(k_1+1)(k_2+1)}{12}(k_1^2+k_2^2+3k_1k_2-k_1-k_2) = D^{(2|1)}_{k_1,k_2|k_1+k_2}$$

- will see that the conformal inv. + polynomial assumption on indices ensure

$$A^{(m|1)}_{k_1,\ldots,k_m|k_1+\ldots+k_m} = D^{(m|1)}_{k_1,\ldots,k_m|k_1+\ldots+k_m} \ , \quad A^{(1|n)}_{\ell_1+\ldots+\ell_m|\ell_1,\ldots,\ell_n} = D^{(1|n)}_{\ell_1+\ldots+\ell_m|\ell_1,\ldots,\ell_n}$$

- The conformal inv. does not control the cases min $(m,n) \geq 2$ with indices $k_i+1, \ \ell_i+1 > 2$ and we find $A^{(2|2)} \neq D^{(2|2)}$ analytically

ii) $\underline{D_{\sqcap}}$

- Representation of a generic coeff $D^{(m|n)}$ as a multiple sum

★ $D^{(m;n)}_{k_1,\ldots,k_m|\ell_1,\ldots,\ell_n} = \text{symmetrized}\left(\sum_{i_1=0}^{k_1}\cdots\sum_{i_m=0}^{k_m}\sum_{j_1=0}^{\ell_1}\cdots\sum_{j_n=0}^{\ell_n}(k_m-i_m)(\ell_n-j_n)\right)$

where $\displaystyle\sum_{p=1}^{m} i_p + \sum_{q=1}^{n} j_q = \sum_{p=1}^{m} k_p = \sum_{q=1}^{n} \ell_q$

- all of $D^{(m|1)}$ can be summed to give

$$D^{(\cdot|1)} \equiv \sum_{m=1}^{\infty} D^{(m|1)}_{k_1,\ldots,k_m|k_1+\ldots+k_m} h_{k_1+1}\cdots h_{k_m+1}\bar{h}_{k_1+\ldots+k_m+1} = -\frac{1}{6}\oint \bar{h}(\bar{\zeta}) S_\zeta(z)\zeta^2 d\zeta$$

$z = \zeta + \sum_k h_k \zeta^k$ and $S_\zeta(z) = \dfrac{z'''}{z'} - \dfrac{3}{2}\left(\dfrac{z''}{z'}\right)^2$; Schwarzian derivative

- Highlight

- coincidence

$$A_i^{(1|1)} = \frac{i(i^2-1)}{6} = D_i^{(1|1)}$$

$$A_{ij}^{(2|1)} = \frac{(i+1)(j+1)}{12}\Big(i^2+j^2+3ij-i-j\Big) = D_{ij}^{(2|1)}$$

$$A_{ijk}^{(3|1)} = \frac{(i+1)(j+1)(k+1)}{18}\Big(i^2+j^2+k^2+3(ij+jk+ik)-(i+j+k)\Big) = D_{ijk}^{(3|1)}$$

- difference

$$\begin{aligned}
A_{ij|kl}^{(2|2)} = \; & \delta_{i+j,k+l}\frac{k+1}{48(i+j-1)(i+j+1)} \\
& \times\Big\{2ij(i^4+5i^3j+8i^2j^2+5ij^3+j^4)+2(i+j)^5-2(k^2-k+1)i^2j^2 \\
& +k^2(k^2+k-2)(i^2-ij+j^2)+(3k^4+3k^3-10k^2+4k-2)ij-k^2(k^2+k-2) \\
& -\frac{1}{i+j}\Big(2(k^3+k^2-2k+2)(i^4+j^4)+(7k^3+9k^2-16k+16)ij(i^2+j^2)+(9k^3+15k^2-24k+24)i^2j^2 \\
& -2(k^3+k^2-2k+1)(i^2+j^2)-(5k^3+3k^2-8k+4)ij\Big)\Big\}\times h_{i+1}h_{j+1}\bar{h}_{k+1}\bar{h}_{l+1}
\end{aligned}$$

$$\begin{aligned}
D_{ij|kl}^{(2|2)} = \; & \delta_{i+j,k+l}\frac{1}{24}\Big((i+1)(j+1)(k+1)(i^2+3ij+j^2-i-j) \\
& -(i+j+2)(k+2)(k+1)k(k-1)+\frac{3}{5}(k+3)(k+2)(k+1)k(k-1)\Big)h_{i+1}h_{j+1}\bar{h}_{k+1}\bar{h}_{l+1}
\end{aligned}$$

iv) Conformal Invariance

With the AdS3 ansatz, reduces to SL(2).
When acting on a functional F[z(s)] of a parametrized curve

$\Pi : s \to C$

The three generators are

$$\hat{J}_- F = \oint \frac{\delta F}{\delta z(s)} ds \ , \qquad \hat{J}_0 F = \oint z \frac{\delta F}{\delta z(s)} ds$$

$$\hat{J}_+ F = \oint z^2 \frac{\delta F}{\delta z(s)} ds$$

$$\hat{J}_- = \frac{\partial}{\partial h_0} \ , \qquad \hat{J}_0 = \frac{\partial}{\partial h_1} + \sum_{k=0}^{\infty} h_k \frac{\partial}{\partial h_k}$$

$$\hat{J}_+ = \frac{\partial}{\partial h_2} + 2 \sum_{k=0}^{\infty} h_k \frac{\partial}{\partial h_{k+1}} + \sum_{k,m=0}^{\infty} h_k h_m \frac{\partial}{\partial h_{k+m}}$$

- the integrand of D_Π varies by a total derivative under infinitesimal version of SL(2,R) ; $z \to \dfrac{az+b}{cz+d}$

- SL(2,R) is an isometry within our ansatz

- Use :

<u>e.g.</u> $A_{ijk}^{(3|1)}$ $\hat{J}_-$ annih. $\Rightarrow$ vanishes if any one of the indices is minus one

with $A^{(2|1)}$, $A^{(1|1)}$ given

$\therefore$ Let

$$A_{ijk}^{(3|1)} = \alpha(i+1)(j+1)(k+1)\Big(i^2+j^2+k^2+\beta(ij+jk+ik)+\gamma(i+j+k)+\delta\Big)$$

$\hat{J}_0$, $\hat{J}_+$ annih. $\Rightarrow$ $\alpha = \dfrac{1}{18}$, $\beta = 3$, $\gamma = -1$, $\delta = 0$

- Hence, the assumptions of the polynomial structure and of the conformal inv. of $A_\sqcap$ and $D_\sqcap$ $\Rightarrow$ $A^{(m|1)} = D^{(m|1)}$

$A^{(1|m)} = D^{(1|m)}$

- $A^{(n|m)}$ with min $(m, n) \geq 2$ fails to be a polynomial.

v) Nonplanar Case

relax the requirement $y^0 = 0$

- the b.c. are now

$$r(\zeta,\bar{\zeta})\Big|_{\zeta\bar{\zeta}=1} = 0$$

$$y_0(\zeta,\bar{\zeta})\Big|_{\zeta\bar{\zeta}=1} = \sum_{k=0}^{\infty} q_k \zeta^k + \sum_{k=1}^{\infty} q_{-k}\bar{\zeta}^k$$

- the iterative procedure

$r(\zeta,\bar{\zeta})$ as before

$$y_0(\zeta,\bar{\zeta}) = b^{(1)}(\zeta,\bar{\zeta}) + b^{(2)}(\zeta,\bar{\zeta}) + \cdots$$

$$b^{(1)}(\zeta,\bar{\zeta}) = \sum_{k=0}^{\infty} q_k \zeta^k g_k(\zeta\bar{\zeta}) + \sum_{k=1}^{\infty} q_{-k}\bar{\zeta}^k g_k(\zeta\bar{\zeta})$$

- A and D coincide up to the cubic order in h, $\bar{h}$ and q .

V)

- progress on the construction of classical spyky solutions in AdS space,

Jevicki, Jin 0903.3389

Dorn, Jordadze, Wuttke 0903.0977

- discretized, numerical approach to NG eq. & minimal surfaces

Dobashi, Ito, Iwasaki 0901.3046, 0805.3594

- progress on the remainder fn

ABHKST, 0902.2245 , 2-loop , $\forall_n$

- exact determination of the remainder function in $R^{1,1}$

Alday-Maldacena 4,5 0903.4707, 0904.0663

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