Spherically symmetric solutions in Massive Gravity and the Vainshtein Mechanism

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based on arXiv:0901.0393, arXiv:0905:2943, work in progress

MOTIVATION

Modification of gravity - a way to get acceleration of the Universe.

Basic Idea: give a mass to a graviton with $m \sim H_0$

Pathologies of Pauli-Fierz non-linear Massive Gravity:

- Hamiltonian unbounded from below (ghosts)
- Singular solutions (?)

However:

- Other models with massive gravitons assume the Vainshtein mechanism to work (E.g. Nair, Randjbar-Daemi, V. Rubakov'08).
- PF MG can be seen as a relatively simple toy model (basic ingredient for models with extra-dimensions like DGP)

Pauli-Fierz Massive Gravity

- Quadratic theory of MG   Fierz'39; Fierz&Pauli'39

$$H_{\mu\nu} = g_{\mu\nu} - f_{\mu\nu}$$

$$S = \frac{M_P^2}{2}\int d^4x \, \left(\text{“}H\partial^2 H + ...\text{''} - \frac{m^2}{4}[H_{\mu\nu}H^{\mu\nu} - (H^\mu_\mu)^2] \right) + \int d^4x \, \frac{1}{2} T_{\mu\nu} H^{\mu\nu}$$

Pauli-Fierz Massive Gravity

✦ Quadratic theory of MG

Fierz'39; Fierz&Pauli'39

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$$S = \frac{M_P^2}{2}\int d^4x \, \Big(\underbrace{\text{“}H\partial^2 H + ...\text{''}}_{\text{Kinetic term}} - \frac{m^2}{4}\underbrace{[H_{\mu\nu}H^{\mu\nu} - (H^\mu_\mu)^2]}_{\text{PF mass term}} \Big) + \underbrace{\int d^4x \, \frac{1}{2}T_{\mu\nu}H^{\mu\nu}}_{\text{Coupling to matter}}$$

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✦ Nonlinear theory

$$S = \int d^4x \, \left(\frac{M_P^2}{2} \sqrt{-g}\, R[g] + \mathcal{V}_{\rm int}[f, g] + \sqrt{-g}\, \mathcal{L}_m[g] \right)$$

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Kinetic term

PF mass term

Coupling to matter

✦ Nonlinear theory

$$S = \int d^4x \, \left(\frac{M_P^2}{2}\sqrt{-g}\, R[g] + \mathcal{V}_{\rm int}[f,g] + \sqrt{-g}\, \mathcal{L}_m[g] \right)$$

Examples:

$$\mathcal{V}^{(BD)}_{\rm int} = -\frac{1}{8}m^2 M_P^2 \sqrt{-f}\, H_{\mu\nu}H_{\sigma\tau}\left(f^{\mu\sigma}f^{\nu\tau} - f^{\mu\nu}f^{\sigma\tau}\right)$$

Boulware, Deser'72

$$\mathcal{V}^{(AGS)} = -\frac{1}{8}m^2 M_P^2 \sqrt{-g}\, H_{\mu\nu}H_{\sigma\tau}\left(g^{\mu\sigma}g^{\nu\tau} - g^{\mu\nu}g^{\sigma\tau}\right)$$

Arkani-Hamed, Georgi, Schwartz'03

Static spherically symmetric solutions

$$S \propto \int d^4x \sqrt{-g}\, R + \mathcal{V}_{\rm int}[\tilde{g}, g]$$

$$R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$$

Vainshtein'72

Non-perturbative regime,
General Relativity

linear regime,
non-General Relativity

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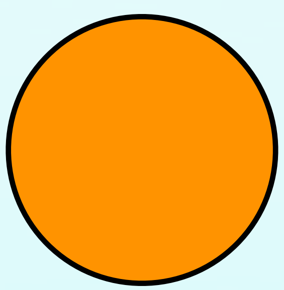

Non-perturbative regime,
General Relativity

linear regime,
non-General Relativity

➡ Is it possible to find a solution regular everywhere?

No Damour, Kogan, Papazoglou'03

Stuckelberg mechanism and decoupling limit

Arkani-Hamed, Georgi, Schwartz'03

- Massive spin-2 graviton: in the action

$$S = \frac{M_P^2}{2}\int d^4x\,\left(\sqrt{-g}R[g] - \frac{m^2}{4}\mathcal{V}\left[\mathbf{g}^{-1}\mathbf{f}\right]\right) + S_m[g],$$

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introduction of Stuckelberg boson

$\Downarrow$ demixing of kinetic terms

$$S \supset \int d^4x \Big\{ M_P^2 \hat{h}\Box\hat{h} + ... + M_P^2 m^2 A\Box A + ... + M_P^2 m^4 \phi\Box\phi + ... \Big\}$$

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✦ Dominant higher order term: $\dfrac{(\partial^2\tilde{\phi})^3}{\Lambda^5}$ with $\Lambda=\left(m^4M_P\right)^{1/5}$

Decoupling Limit

$$\boxed{\begin{array}{rcl} M_P & \rightarrow & \infty \\ m & \rightarrow & 0 \\ \Lambda & \sim & const \\ T_{\mu\nu}/M_P & \sim & const \end{array}}$$

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Can do the same in equations of motion applied to the spherically symmetric configurations

Action for $\tilde{\phi}$ in the Decoupling limit

- The action for the scalar sector:

$$S = \frac{1}{2}\int d^4x \left\{ \frac{3}{2}\tilde{\phi}\Box\tilde{\phi} + \frac{1}{\Lambda^5}\left[\alpha\,(\Box\tilde{\phi})^3 + \beta\,(\Box\tilde{\phi}\;\tilde{\phi}_{,\mu\nu}\;\tilde{\phi}^{,\mu\nu})\right] - \frac{1}{M_P}T\tilde{\phi} \right\}$$

Spherically Symmetric case:

$$3\,\frac{\tilde{\phi}'}{R} \quad + \quad \frac{2}{\Lambda^5}\left\{3\alpha\,\left(-4\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 2\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + \frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right) + \right.$$
$$\left. +\beta\left(-6\frac{\tilde{\phi}'^2}{R^4} + 2\frac{\tilde{\phi}'\tilde{\phi}''}{R^3} + 4\frac{\tilde{\phi}''^2}{R^2} + 2\frac{\tilde{\phi}'\tilde{\phi}^{(3)}}{R^2} + 3\frac{\tilde{\phi}''\tilde{\phi}^{(3)}}{R}\right)\right\}$$
$$= -\frac{1}{R^3}\int_0^R d\tilde{R}\;\tilde{\rho}\left(\tilde{R}\right)\;\tilde{R}^2$$

✦ New rescaled variables: $\xi \equiv \dfrac{R}{R_V} \quad , \quad w \equiv -\Lambda^5 R_V^2 \times 2\dfrac{\tilde{\phi}'}{R}$

$$\boxed{2\,Q(w) + \frac{3}{2}w = \frac{1}{\xi^3}}$$

$$Q(w) = -\frac{1}{2}\left\{3\alpha\left(\frac{\xi}{2}\dot{w}\ddot{w} + \frac{3}{2}w\ddot{w} + 2\dot{w}^2 + \frac{6w\dot{w}}{\xi}\right)\right.$$
$$\left. + \beta\left(\frac{3\xi}{2}\dot{w}\ddot{w} + \frac{5}{2}w\ddot{w} + 5\dot{w}^2 + \frac{10w\dot{w}}{\xi}\right)\right\}.$$

$$\boxed{2Q(w) + \frac{3}{2}\, w = \frac{1}{\xi^3}}$$

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close to source

Vainshtein solution,

$$2Q(w) + \cancel{\frac{3}{2} w} = \frac{1}{\xi^3}$$

$$w \propto \frac{1}{\sqrt{\xi}}, \; (\nu = -\lambda)$$

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perturbative regime,

$$\cancel{2Q}(w) + \frac{3}{2}\,w = \frac{1}{\xi^3}$$

$$w \to \frac{2}{3\xi^3},\ (\nu = -2\lambda)$$

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perturbative regime,

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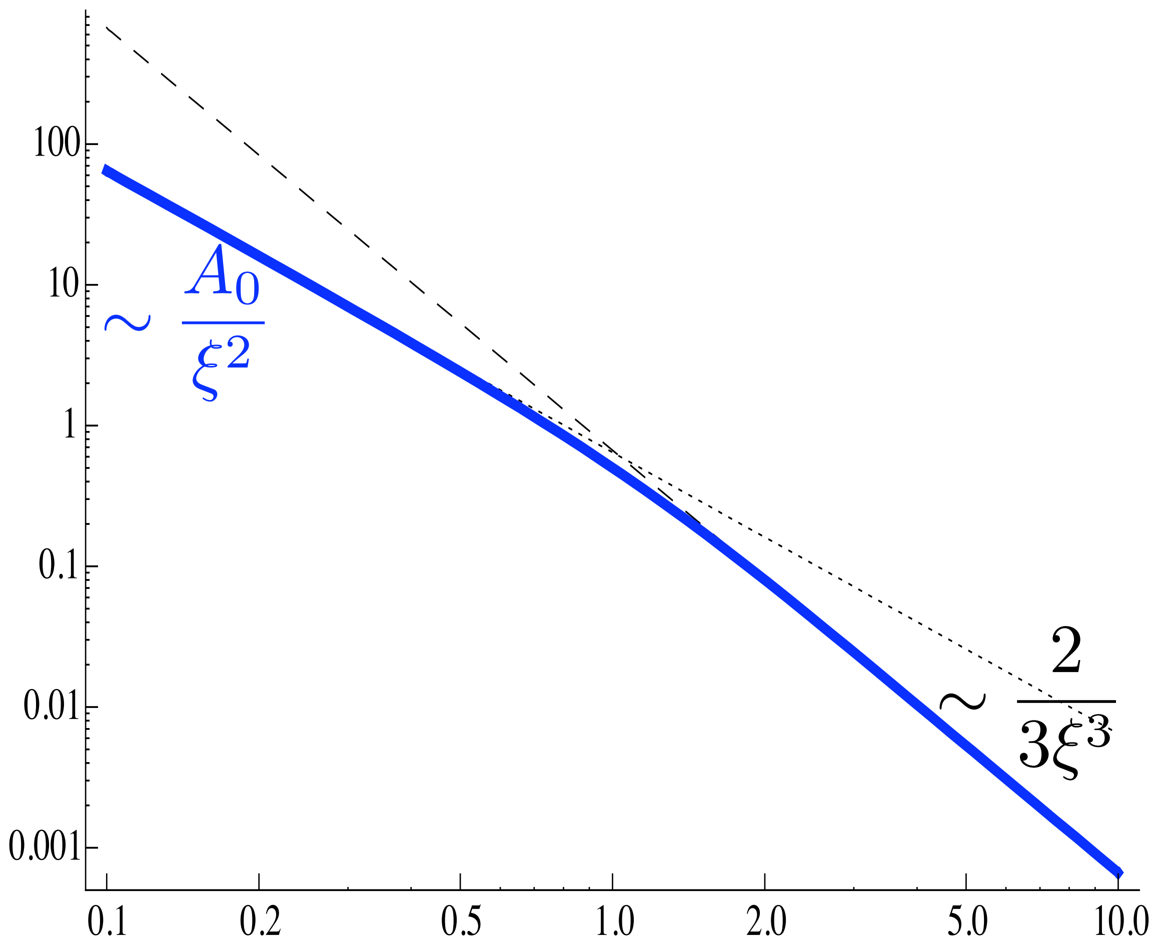
Another solution!

$$2Q(w) + \cancel{\frac{3}{2}w} = \cancel{\frac{1}{\xi^3}}$$

$$Q(w) = 0 \Rightarrow w \propto \xi^{p_{1,2}}, \ (\nu = -\lambda)$$

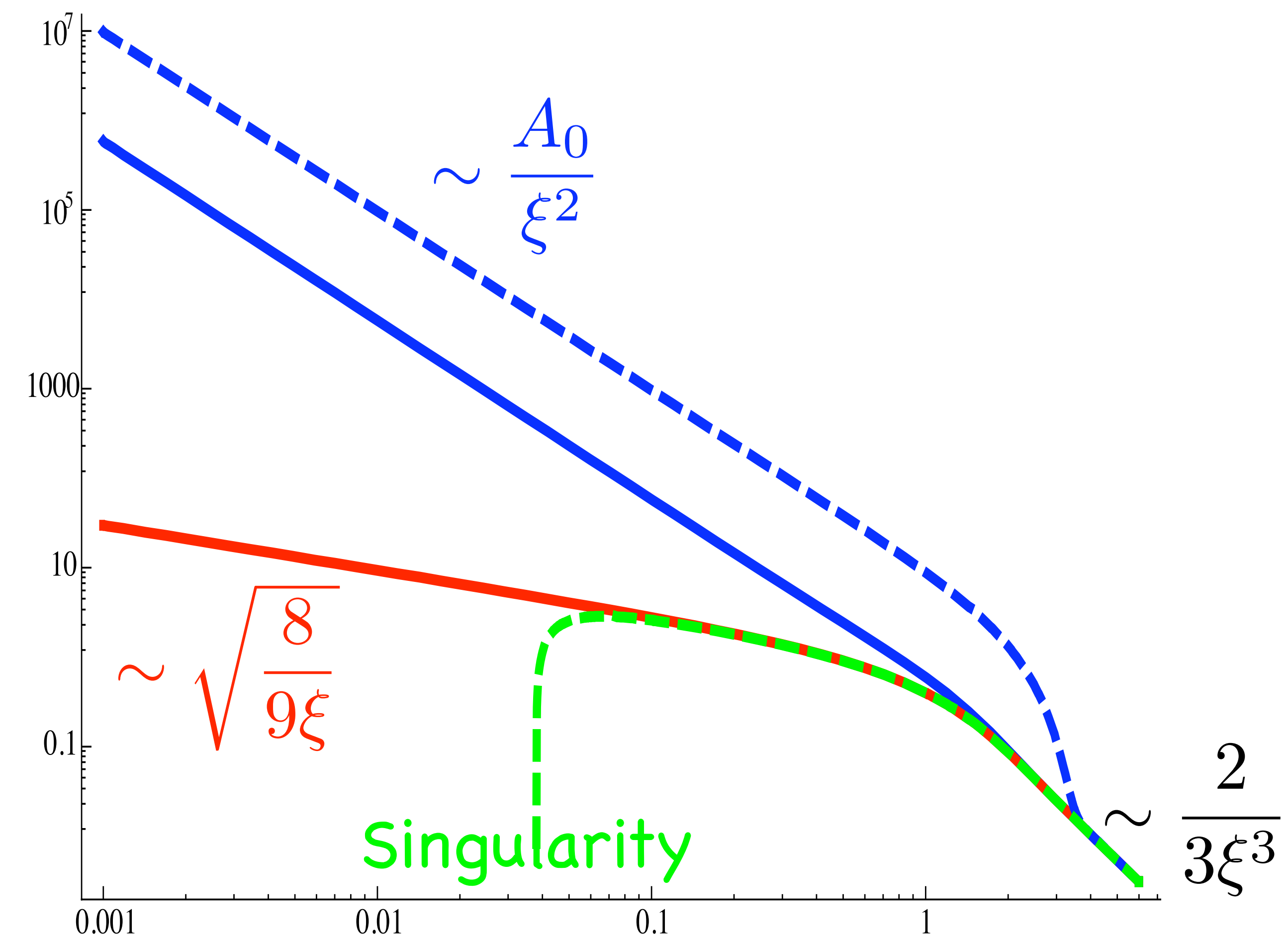
Solutions in the Decoupling Limit

BD potential



Unique solution for the
fixed flat asymptotic at infinity

AGS potential

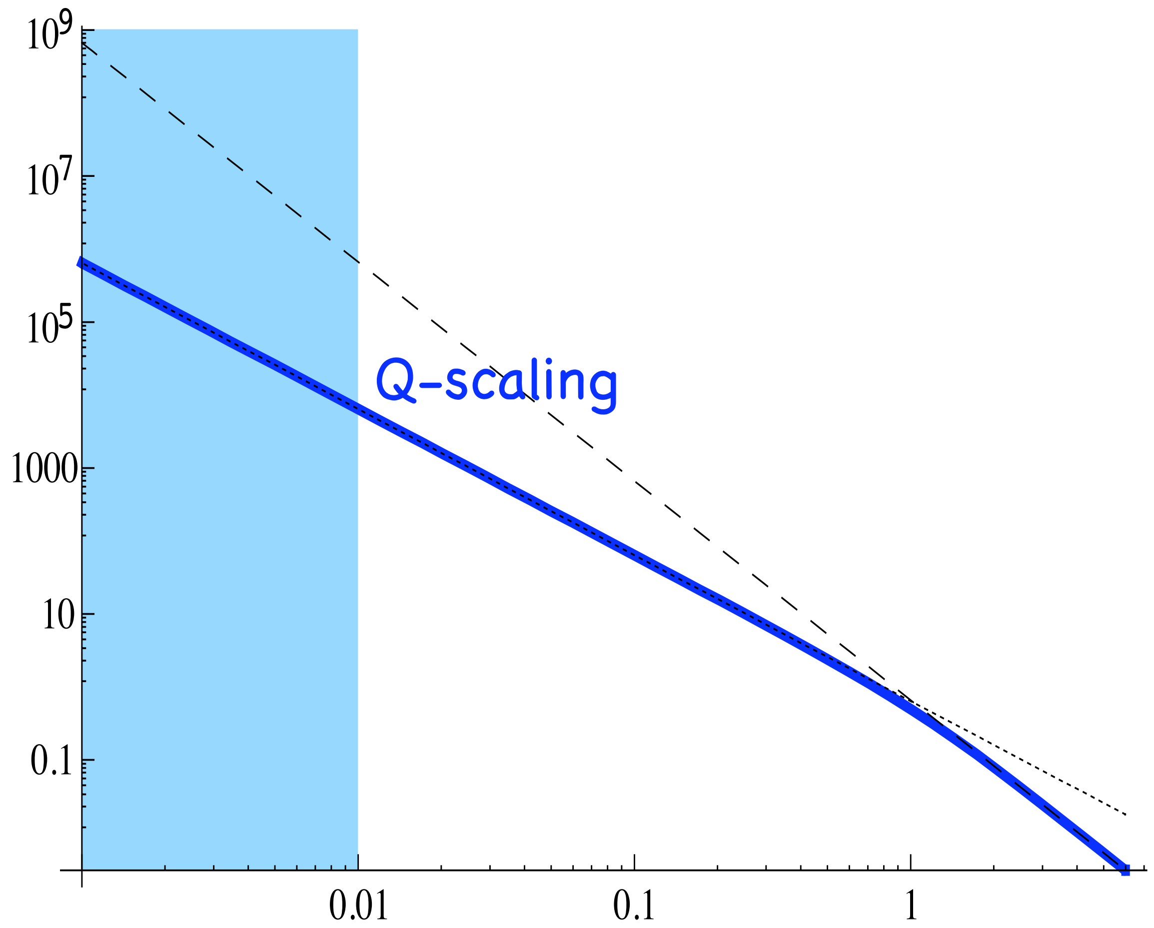


A family of solutions with the
same flat asymptotic at infinity

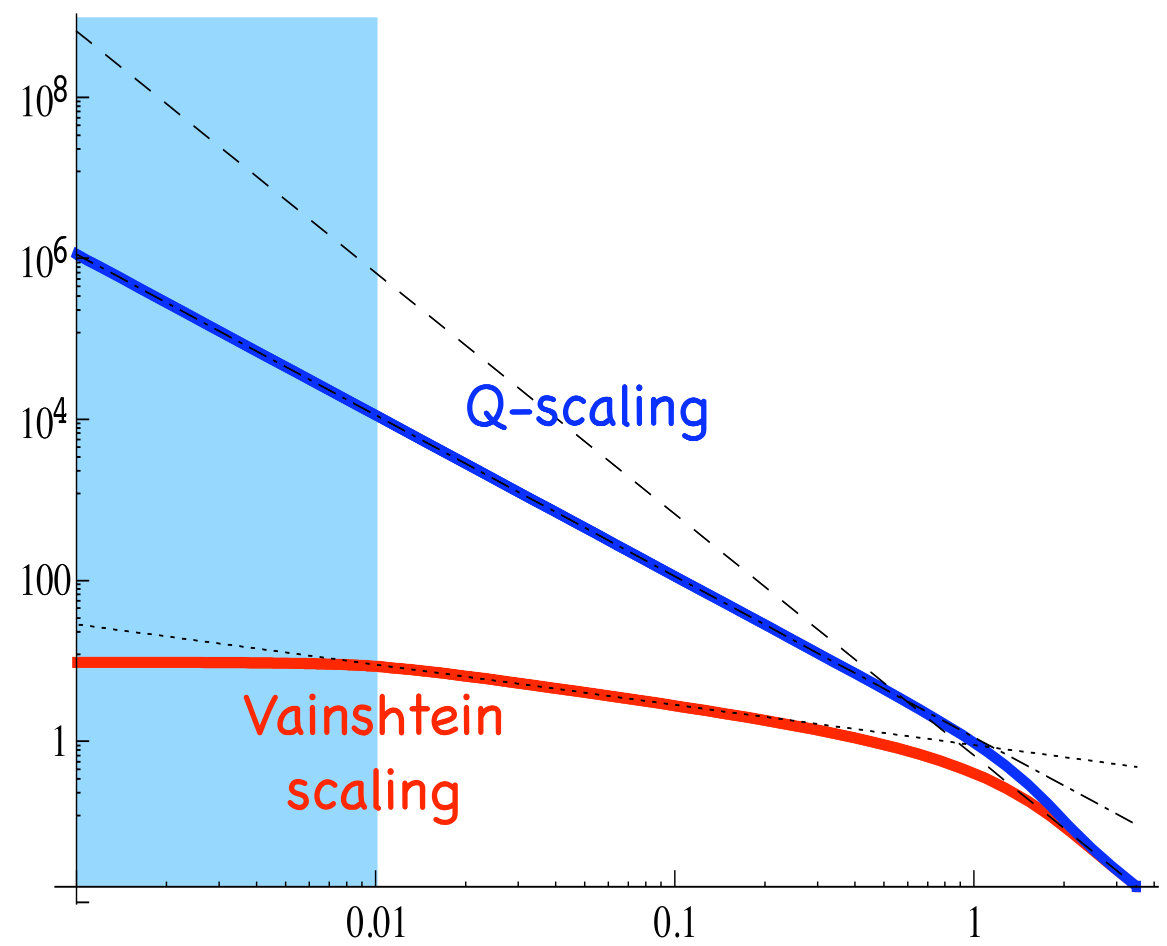
solutions with source

- Let us include a smoothed source and ask for regularity at r=0

BD potential



AGS potential



Full system: Metrics and Equations of Motion

$$g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(R)}dt^2 + e^{\lambda(R)}dR^2 + R^2 d\Omega^2$$ Schwarzschild-like

$$f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2 d\Omega^2$$ flat

- Equations of motion:

$$\begin{aligned}
e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) &= 8\pi G_N\left(T^g_{tt} + \rho e^\nu\right), \\
\frac{\nu'}{R} + \frac{1}{R^2}\left(1 - e^\lambda\right) &= 8\pi G_N\left(T^g_{RR} + P e^\lambda\right), \\
\nabla^\mu T^g_{\mu R} &= 0.
\end{aligned}$$

- Relation between μ and ϕ

$$\phi' = -\frac{R\mu}{2}$$

Different asymptotic solutions

large R:

$$\begin{aligned}
\lambda_0 &= \frac{mC_1}{2}\left(1+\frac{1}{mR}\right)e^{-mR}, \\
\nu_0 &= -\frac{C_1}{R}e^{-mR}, \\
\mu_0 &= \frac{C_1}{2R}\left(1+\frac{1}{mR}+\frac{1}{(mR)^2}\right)e^{-mR}
\end{aligned}$$

not so large R:

$$\begin{aligned}
\nu &= -\frac{2}{3}\frac{R_S}{R}+\frac{R_S^2}{R^2}\frac{n_1}{(mR)^4}+\mathcal{O}(R_S^3) \\
\lambda &= \frac{1}{3}\frac{R_S}{R}+\frac{R_S^2}{R^2}\frac{l_1}{(mR)^4}+\mathcal{O}(R_S^3) \\
\mu &= \frac{1}{3(mR)^2}\frac{R_S}{R}+\frac{R_S^2}{R^2}\frac{m_1}{(mR)^6}+\mathcal{O}(R_S^3)
\end{aligned}$$

assume $R \gg R_S$

Vainshtein scaling (small R)

$$\nu = -\frac{R_S}{R} + n_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right)$$

$$\lambda = \frac{R_S}{R} + l_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}\left(m^4\right)$$

$$\mu = m_0 \sqrt{\frac{R_S}{R}} + m_1 (mR)^2 + \mathcal{O}\left(m^4\right)$$

$$\boxed{R_V = \left(\frac{R_S}{m^4}\right)^{1/5}}$$

solutions very far from source:
infinitely many solutions!

Decoupling Limit

$$\mu \sim R^{-3} + \ldots$$

$$\delta\mu \sim Ce^{-\#R}$$

Full system

$$\mu \sim e^{\#-R} + \ldots$$

$$\delta\mu \sim Ce^{-\#e^{\#\sqrt{R}}}$$

$\boxed{\text{Important for numerics!!!}}$

Validity of DL solutions

All other terms <-> cubic interaction (kept in DL)

- for R>(Vainshtein radius) DL is valid up to 1/m
- for R<(Vainshtein radius),

Q-scaling

$$h \sim R_S/R$$
$$\partial\partial\phi \sim \mu \sim m^2 R_V^4/R^2$$
$$A \sim 0$$
$$\Downarrow$$
$$R \sim R_V^2 m$$

$$R_V^2 m \ll R \ll m^{-1}$$

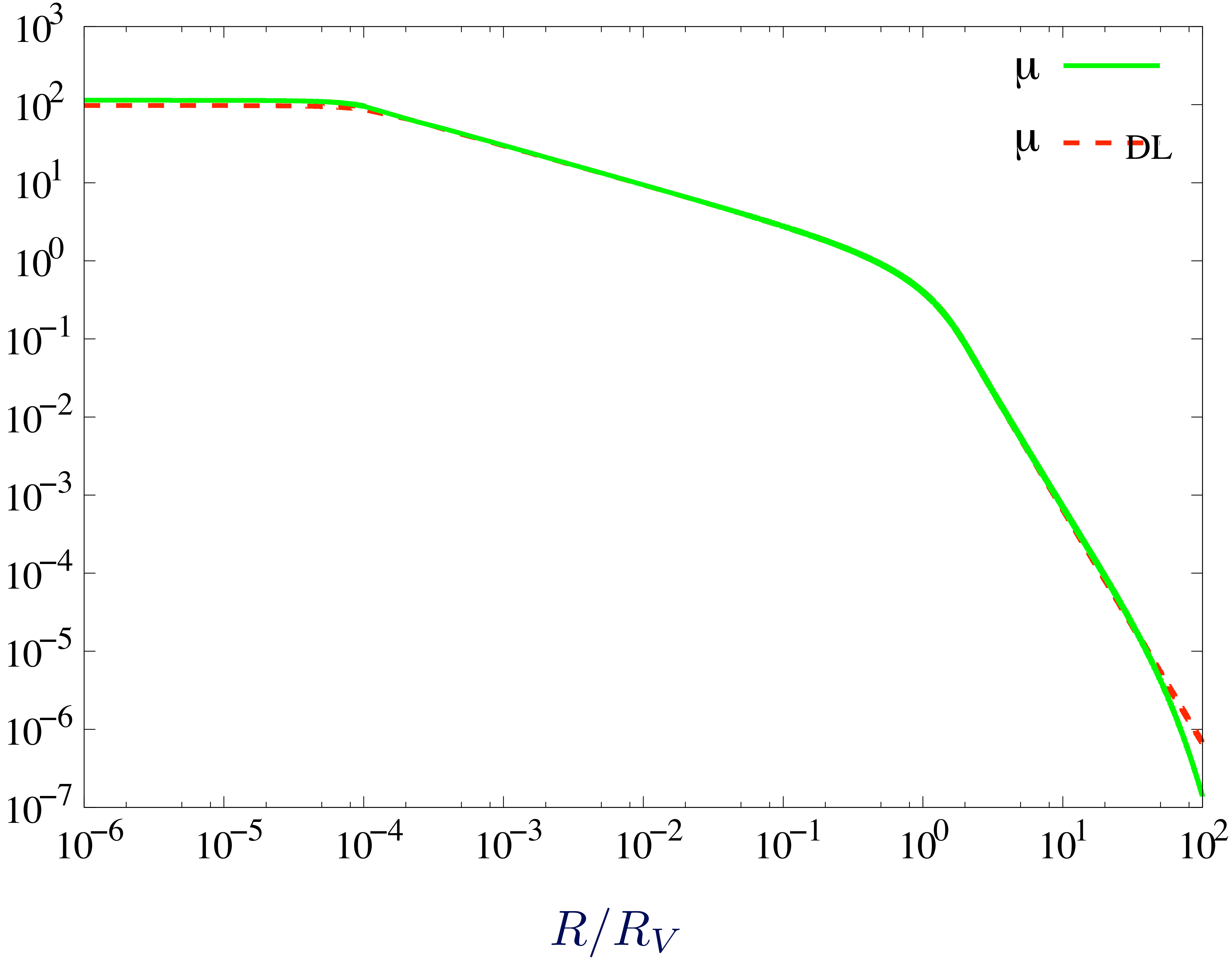
Vainshtein scaling

$$h \sim R_S/R$$
$$\partial\partial\phi \sim \mu \sim \sqrt{R_S/R}$$
$$A \sim 0$$
$$\Downarrow$$
$$R \sim R_S$$

$$R_S \ll R \ll m^{-1}$$

N.B. Inside the star the solution changes,
DL is still valid.

Decoupling Limit <-> full system



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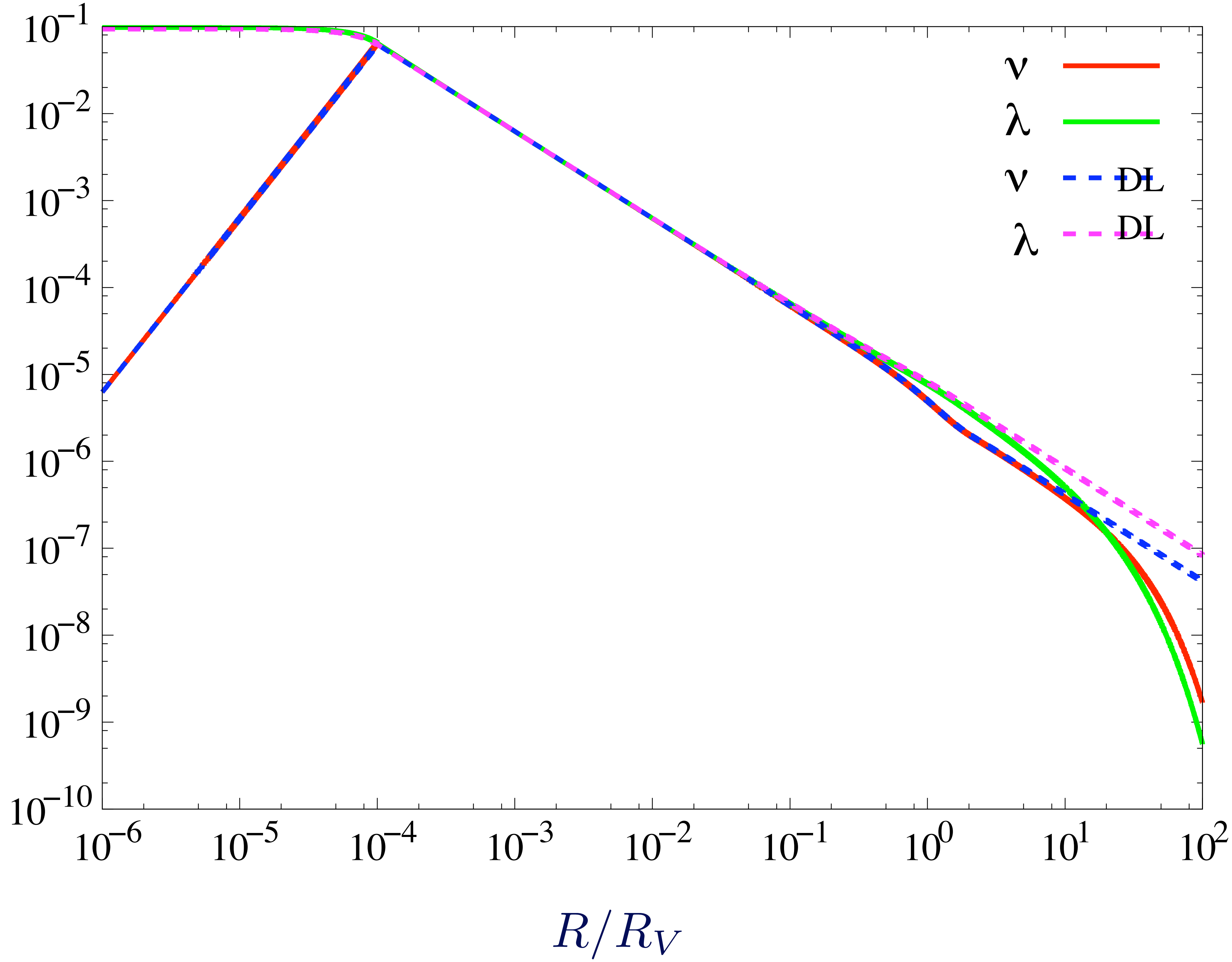
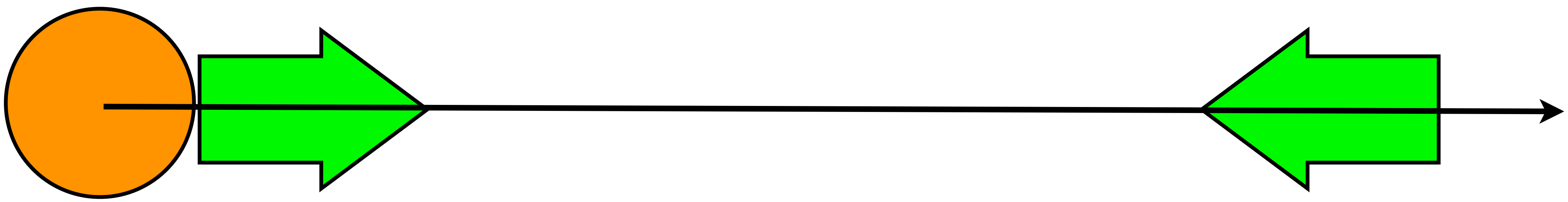


Numerics

RELAXATION vs SHOOTING



Damour, Kogan, Papazoglou'03

Relaxation

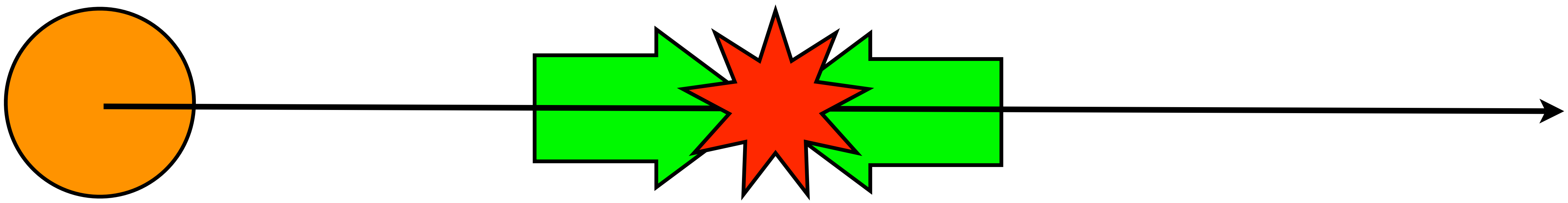
Impose all the boundary conditions
Might miss a singularity

Shooting

More reliable for checking singular solutions
Requires adjusting initial conditions to get required boundaries
Extremely difficult for highly non-linear systems and for several equations

Numerics

RELAXATION vs SHOOTING



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from MG to k-Mouflage Gravity

✦ general class of actions

$$S = M_P^2 \int d^4x \sqrt{-g}\left(R + m^2 \phi R + m^2 F(\phi)\right) + S_m,$$

$$S = M_P^2 \int d^4x \left(\text{"} h\Box h\text{"} + m^2 \phi\text{"}\Box h\text{"} + m^2 F(\phi)\right) + S_m$$

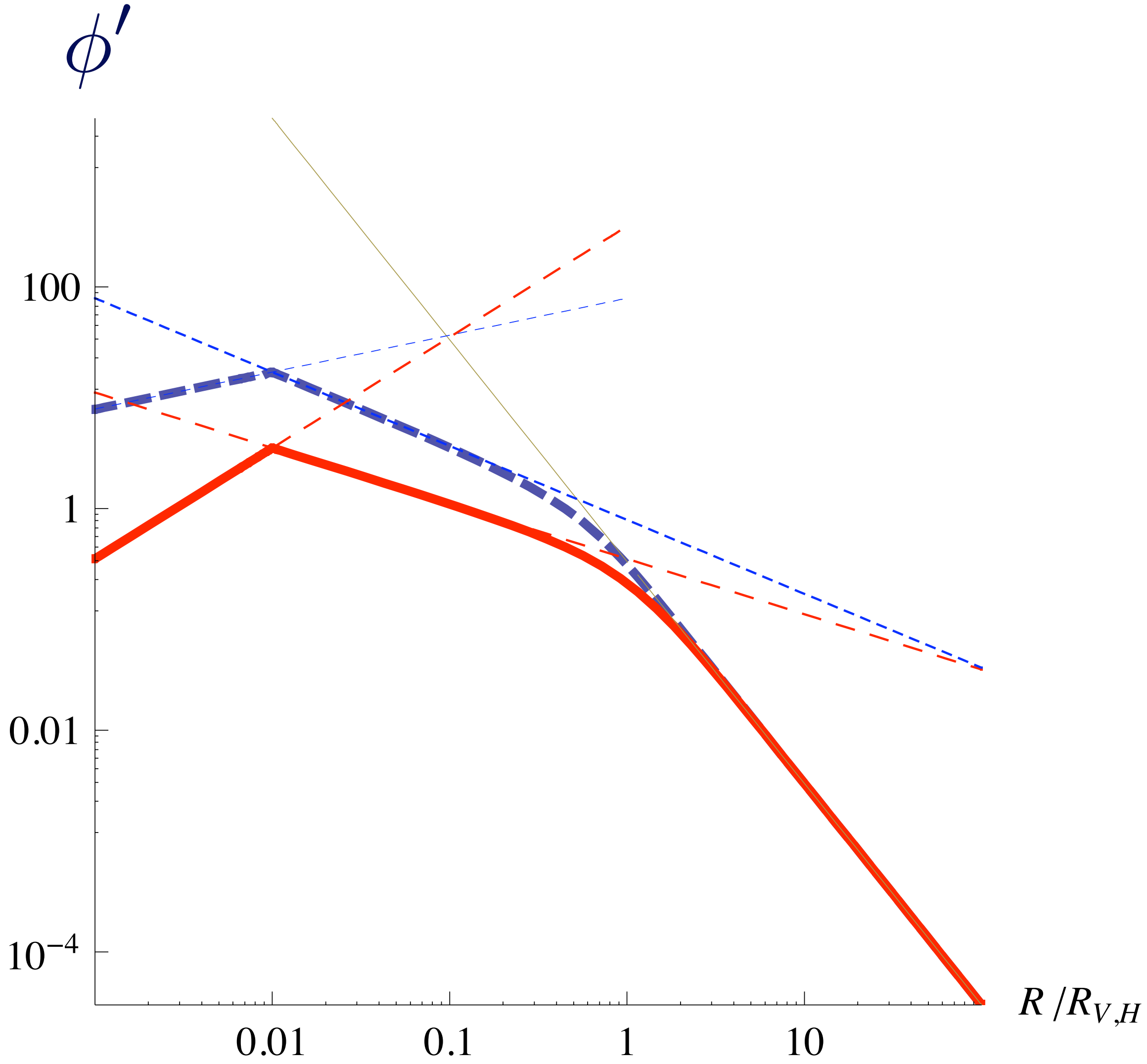
✦ equations of motion

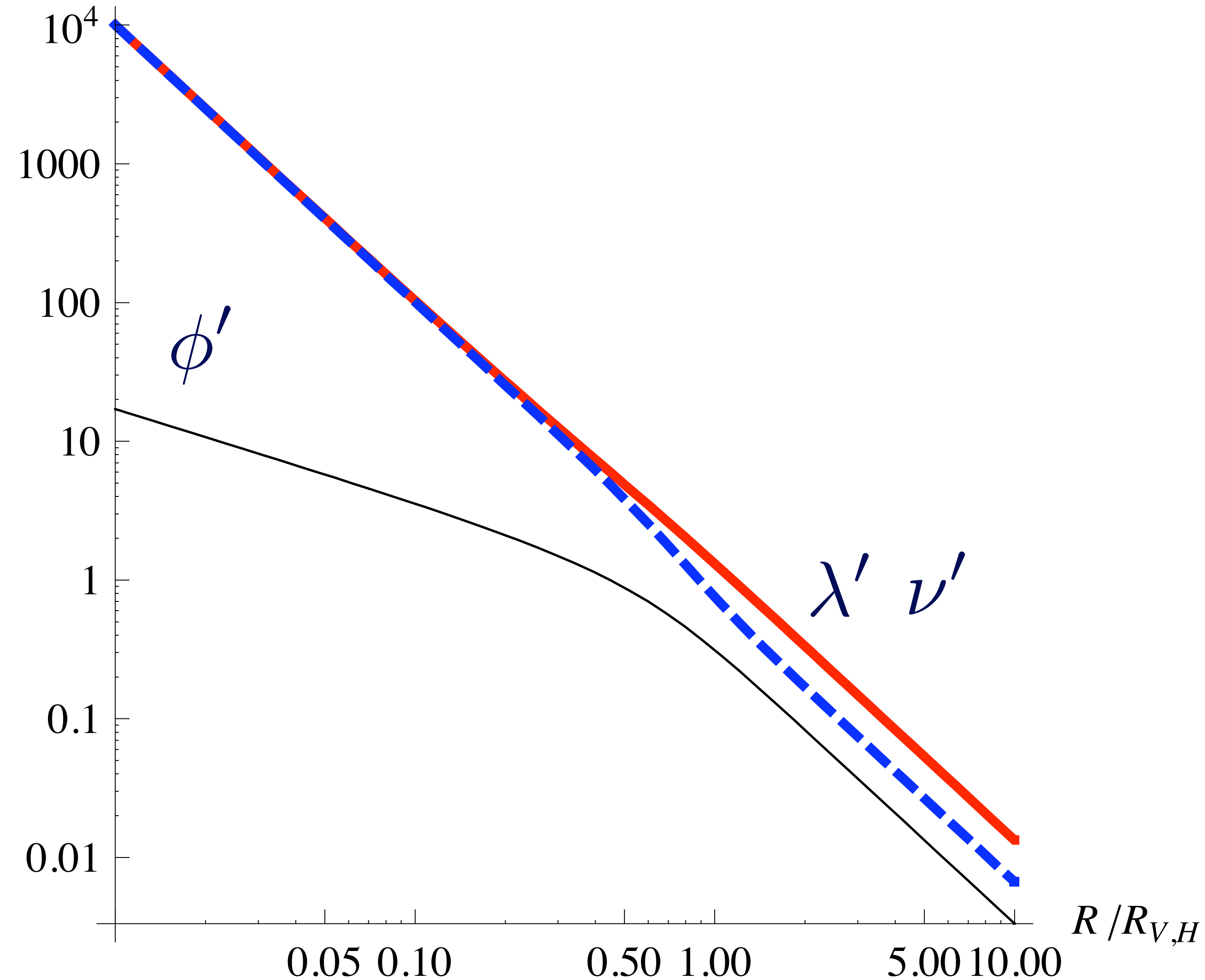
$$\text{"}\Box h\text{"} + m^{2}\text{"}\Box\phi\text{"} = 0$$

$$\text{"}\Box h\text{"} + \frac{\delta F(\phi)}{\delta\phi} = 0$$

=> $$m^{2}\text{"}\Box\phi\text{"} + \frac{\delta F(\phi)}{\delta\phi} = 0$$

from MG to k-Mouflage Gravity





Conclusion (I)

- It is possible to obtain the DL in the case of static spherically symmetric ansatz. This decoupling limit corresponds to DL in the Goldstone picture.
- The scaling conjectured by Vainshtein at small radius is only a limiting case in an infinite family of non singular solutions each showing a Vainshtein recovery of GR solutions below the Vainshtein radius but a different common scaling at small distances.
- For AGS potential a family of solutions exists containing the new scaling solution with an Vainshtein-like solution as an asymptotic. The requirement of no-conical singularity at zero chooses uniquely the Vainshtein-like solution.
- For the full system (not DL) regular (everywhere) solution exist for AGS potential featuring a Vainshtein-like recovery of solutions of General Relativity and flat asymptotic at infinity.
- ? Compact objects: neutron stars and black holes ?

Conclusion (II)

- A large class of scalar-tensor theories where gravity becomes stronger at large distances via the exchange of a scalar that mixes with the graviton.
- At small distances, i.e. large curvature, the scalar is screened ("camouflages") via an analog of the Vainshtein mechanism of massive gravity.